Mode of Transport along the Quality Dimension

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Abstract

Aeroplanes are a fast but expensive means of shipping goods, which allow producers to respond quickly to favourable demand realisations and to limit the risk of shipping unprofitably large quantities during low demand periods while substantially raising the transportation cost relative to ocean cargoes. We explore the role of heterogeneous income elasticity of demand faced by exporters of quality-differentiated goods in shaping the transport mode choice. We find that more considerable demand volatility induces more exporters to opt for air shipping, and more so if they produce high-quality goods. We also produce supporting evidence based on U.S. data at the exporter-district-product level.

JEL Classification: F1; F14

Keywords: International trade; Transportation mode; Unit values; PIGL utility function

1 Introduction

Two major factors influencing the choice of mode of transportation for international trade flows are the cost per distance and the timeliness of delivery. The trade-off between these two factors appears most pronounced when comparing air and sea freight. Air transportation offers a substantially faster option than the sea, albeit at a much higher cost. For some industries, the intrinsic characteristics of

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1For the case of the U.S., which is the country we will focus on as an importer in this paper, the vast majority of imports are shipped either by air or sea, except for those originating from Canada and Mexico. When excluding Canada and Mexico, the share of U.S. imports (in value) shipped by either air or sea in 2017 was 96% (computed as the average across all exporters to the US, except Canada and Mexico).
their products are what ultimately determine their choice. Some goods have a relatively short lifespan before drastically losing value and hence need to reach their destination market fast. Other goods are too heavy per unit or value for air freight to be economically feasible. Nevertheless, as shown in Hummels and Schaur (2010), bilateral trade flows comprise many goods that actively use both air and sea routes. Furthermore, mixing air and sea freight transportation at the product level is widely observed even when considering trade flows between a given origin and destination country.

This paper focuses on products that are actively shipped internationally by both air and sea, even within the same origin-destination pair. What determines the mode of transportation used for trade flows of these products? In Section 2, we document that the specific mode used by products that simultaneously combine air and sea routes tends to correlate with their level of quality (proxied by unit values). In particular, air transportation is used more prominently by higher-quality varieties. This observation suggests that higher-quality varieties of products are those for which enjoying faster delivery is relatively more important. We argue that this pattern of sorting by mode of transportation responds to differences in income elasticities across varieties of different quality.

An extensive literature has recently grown documenting the presence of nonhomotheticities along the quality dimension. This feature of consumer preferences implies that income elasticities will be greater for higher-quality varieties of vertically differentiated products. In the presence of consumer expenditure volatility, higher income elasticity of demand will translate into greater volatility of product sales. As a result, the impact of uncertainty about consumers expenditure becomes increasingly important at higher levels of quality.

One key feature of international trade flows is the longer time gaps mediating output decisions and the moment when the output reaches its destination. Output choices are made based on the expected demand for the product once it reaches its final market. If shipments to a given market cannot be instantly adjusted, in the presence of consumer expenditure uncertainty, (ex-ante) output choices will generally turn out to be (ex-post) suboptimal. This distortion is created by the final demand volatility/uncertainty exporters face when choosing the amount of output to be shipped to a given destination market. As a result, the severity of this distortion increases with two factors: i) time gaps between production and final sale (since longer time gaps increase uncertainty); ii) the level of quality of output (since higher-quality varieties exhibit greater income elasticity).

Our model features several goods, importers with nonhomothetic preferences and exporters supplying two quality-differentiated versions of each good. Demand uncertainty dictates that the transport mode choice be delayed as far as possible in time. Therefore, profit maximising exporters make their choice comparing the profit expectations resulting from air vs sea shipping their goods, taken at the moment that minimises the time lag of seaborne imports. In line with the observations discussed in Section 2, we find that more considerable demand volatility induces more exporters to opt for air

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Related Literature

The technology of transportation modes and, relatedly, transportation costs have evolved quickly over the last two centuries. Hummels (2007) documents the reduction in shipping costs from 1850 to date, focusing on characterising the patterns of international ocean and air transportation costs since World War II. Although some types of goods remain inherently tied to infrequent and lumpy trade (Alessandria, Kaboski and Midrigan, 2010; Hornok and Koren, 2015), for others, the timing of shipping has become a key feature in shaping profitability, volume and quality of production.

Several contributions have looked into the producers’ choice between fast and expensive air shipping and slow and cheap sea shipping. Harrigan (2010) explores the implications of this choice for comparative advantage in a Ricardian framework, finding across U.S. imports a negative relationship between distance and market shares and a positive one between distance and unit values. The mainstream rationalisation of this evidence has been based on productivity aspects, typically building on the classical contribution by Melitz (2003): see, e.g., Baldwin and Harrigan (2011), Whang (2014), Harrigan, Ma and Shlychkov (2015), and Görg, Halpern and Muraközy (2017).

Another strand of the literature has instead explored the influence of another fundamental aspect of transportation mode choice, which springs from the fact that importing from distant locations implies a lag between the shipment and the arrival of a good. Hummels and Schaur (2010) argue that these transit lags generate risks for producers facing volatile demand since they must decide on the quantity to ship before the resolution of demand uncertainty. Aeroplanes are a fast but expensive means of shipping goods, which allow producers to respond quickly to favourable demand realisations and to limit the risk of shipping unprofitably large quantities during low demand periods while substantially raising the transportation cost relative to ocean cargoes. The authors show that U.S. airborne imports correlate positively with the volatility of demand and negatively with the relative cost of air (to sea) shipping. Furthermore, Hummels and Schaur (2013) investigate the role of the price elasticity of demand and the value that consumers attach to fast delivery, estimating a per-day ad-valorem tariff equivalent for goods in transit.

We build on these contributions by studying the effect of nonhomothetic demand in the product-quality dimension on selecting the transport mode. The importance of income-dependent demand for quality is well-documented in the literature of international trade: see, e.g., Bastos and Silva (2010), Feenstra and Romalis (2014), Lugovskyy and Skiba (2016), and Chen, Juvenal and Leigh (2020). In particular, higher-quality goods are associated with greater income elasticity of demand: see, e.g., Manova and Zhang (2012), Fajgelbaum, Grossman and Helpman (2011), and Jaimovich and Merella (2012, 2015). We contribute to the literature by showing how the heterogeneity in demand volatility

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3 These results echo the findings by, e.g., Hummels and Skiba (2004), among others.
that producers of goods of different quality levels face influences the transport mode choice.

2 Dataset and Stylised Facts

This section presents a number of stylised facts concerning the choice of mode of transport for trade flows and how that choice correlates with product quality. In the following sections, we aim at rationalising those stylised facts by introducing a model where vertically differentiated firms optimally choose the mode of transport for their exports, given their output quality and distance to final destination markets.

The stylised facts presented below are based on U.S. import data disaggregated at HS 10-digit level from the U.S. Census Bureau. We restrict the analysis to US imports in the year 2017. The dataset records, amongst other variables, total values, weights and quantities of imports by country of origin and district of entry into the U.S., alongside the mode of transport adopted (air or vessel). There are 42 different districts of entry for U.S. imports recorded in the dataset. We augment the U.S. import dataset with data on sea and air distances from the country of origin to the district of entry of imports. The U.S. Census Bureau data reports the country of origin of imports, but it does not report the port/district of departure of shipments. Consequently, we compute sea and air distances as average distances between the district of entry in the U.S. and the main ports/airports in each country of origin.

For the purpose of our analysis, we trimmed the data along several dimensions. Firstly, to focus on US districts for which total imports shipped by air and by sea are both economically significant, we kept only districts whose total value of seaborne and airborne imports are both greater than 10 million US dollars. In addition, we discarded districts of entry not located in the US mainland. These two trims leave us with 25 different districts of entry (17 districts dropped). Secondly, we removed four sets of exporters: i) landlocked countries, ii) Mexico and Canada (the only two countries sharing a border with the US), iii) countries with a population below 1 million in 2017, iv) countries whose area is greater than 2,500,000 sq km. After these trims, we are left with 83 different countries of origin. Removing the smallest countries avoids dealing with noisy observations in terms of value and quantities. Instead, the largest countries are dropped to mitigate the risk of substantial

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4 Most districts of entry of imports comprise more than one seaport or airport located within the district.
5 To give some sense of this magnitude, the median total value of imports transported by sea across districts of entry is 2.71 billion US dollars, while that of those transported by air is 1.25 billion US dollars.
6 This trim entails dropping Honolulu Harbor (Hawaii), Anchorage (Alaska), and San Juan (Puerto Rico).
7 There are eight countries, besides the US, above this threshold: Russia, Canada, China, Brazil, Australia, India, Argentina, and Kazakhstan. As mentioned above, Canada is already excluded owing to sharing a land border with the US. In addition, Kazakhstan is also excluded as a result of being landlocked. Our results are robust to keeping countries with an area smaller than 7,000,000 sq km (which would amount to keeping India and Argentina in the sample).
measurement error in the distance measures (it is exactly for countries with such a vast area that knowing the port of departure of shipments would be most needed to measure bilateral distances precisely). Finally, given that we intend to study the possibility of selecting a mode of transport along the quality dimension, we restrict the analysis to manufacturing goods, which amount to 10,622 products out of 11,949 HS10 product categories.

Mode of Transport Choice: Sea vs Air Distances

One intriguing stylised fact in the trade literature is the observation that a large number of products sourced from a given exporter are transported by using a mix of air and sea routes – see, e.g., Hummels and Schaur (2010, 2013), Martincus, Carballo and Graziano (2015), Hornok and Koren (2015). In particular, relying on HS10 product-level data, Hummels and Schaur (2010) show that about 35% of product-exporter (PE) combinations were shipped to the US over 1990-2004 by mixing sea and air freight.
Table 1.
Mode of Transport and Relative Sea vs Air Distance.

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rel. distance (sea/air)</td>
<td>-0.331***</td>
<td>-0.261***</td>
<td>-0.431*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.071)</td>
<td>(0.030)</td>
<td>(0.250)</td>
<td></td>
</tr>
<tr>
<td>(log) sea distance</td>
<td></td>
<td></td>
<td>0.158</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.322)</td>
<td></td>
</tr>
<tr>
<td>(log) air distance</td>
<td></td>
<td></td>
<td>-0.065</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.359)</td>
<td></td>
</tr>
<tr>
<td>(log) rel. distance (sea/air)</td>
<td></td>
<td></td>
<td>-0.309***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.036)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>322,948</td>
<td>283,874</td>
<td>283,874</td>
<td>283,874</td>
</tr>
<tr>
<td>Exporter-product FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>District-Product FE</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Note. The dependent variable in all columns is the share of US imports (measured in weight) transported by sea freight. Robust standard errors clustered at exporter-district level. *p < 0.1 **p < 0.05; ***p < 0.01

In Figure 1, we showcase the previous findings presented by Hummels and Schaur (2010) but dig further into the exact point of entry into the US. We do so by separating trade flows at the product-exporter-district (PED) level. The bar graph plots the shares of products per exporter shipped exclusively by air, by sea, or by a mix of transport modes. We further split the observations in which both air and sea transport are used in two separate cases: i) when mode mixing at the PE level is observed only across different districts of entry – the top (red) portion of the bar, labelled as ‘PE’; ii) when mode mixing at the PE level is observed even within the same district of entry – the (bottom) blue portion of the bar, labelled as ‘PED’. For over 37% of PE level combinations, we observe a mix of modes of transport used for shipping to the US. In about two-thirds of those cases, mixing between sea and air transport occurs even within the same US districts (that is, in 24% of trade flows at the PE level, both air and sea transport are simultaneously used to ship products to a given US district).

One advantage of looking into trade flows at the product-by-exporter-by-district-of-entry level is that it allows us to link the mode of transport to differences in relative distance between the sea and air routes from the country of origin to the district of entry. As Hummels and Schaur (2013) highlighted, the East-West coast geography of the US means that sea-vs-air bilateral distances tend to vary quite substantively across different districts of entry located on each of the two coasts. Table 1 exploits variation in sea-vs-air bilateral distances to see how that affects the share of trade
Table 2.
Alchian-Allen Effect by Mode of Transport.

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(log) sea distance</td>
<td>0.093***</td>
<td>0.097***</td>
<td>0.154***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.015)</td>
<td>(0.023)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(log) air distance</td>
<td></td>
<td></td>
<td></td>
<td>0.249***</td>
<td>0.244***</td>
<td>0.270***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.062)</td>
<td>(0.052)</td>
<td>(0.068)</td>
</tr>
<tr>
<td>Observations</td>
<td>322,948</td>
<td>283,874</td>
<td>283,874</td>
<td>283,874</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exporter-product FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>District FE</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>District-Product FE</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Note. The dependent variable in (1)-(3) is the (log) unit value for imports transported by sea. The dependent variable in (4)-(6) is the (log) unit value of imports transported by air. Unit values in all regressions are based on FOB import values. Robust standard errors clustered at exporter-district level. *p < 0.1 **p < 0.05; ***p < 0.01.

flows shipped by sea. As we can observe in column (1), exporters tend to ship larger shares of their products via air routes when sending their goods to US districts whose relative sea-vs-air bilateral distance is greater. Column (2) shows that this correlation is robust to the inclusion of district-product fixed effects; hence it is not driven by certain US districts tending to source a given product by some specific mode of transport. Furthermore, column (4) suggests that the mode of transport responds primarily to differences in relative distances between the alternatives and not variations in absolute distances by either sea or air routes.

Alchian-Allen Effect by Air and Sea: Distance, Quality, and Mode of Transport

The Alchian-Allen effect predicts that exporters will send higher-quality varieties of products to more distant markets. This prediction has been widely supported in the data that exploits cross-country variation in bilateral distances – e.g., Hummels and Skiba (2004), Baldwin and Harrigan (2011). In Table 2, we study whether the Alchian-Allen predictions hold when exploiting variation in distances from a country of origin to different US districts and by separating trade flows according to the mode of transport.

The dependent variable in each of the regressions in Table 2 is the FOB unit value of imports at the HS10 digit, separating trade flows by mode of transport. Columns (1)-(3) regress (log) unit values of imports on the (log) sea distance between the country of origin and district of entry of imports.
Table 3.
Unit Values by Mode of Transport.

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>airborne import</td>
<td>0.881***</td>
<td>0.888***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.018)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>airborne import x</td>
<td>-0.187**</td>
<td>-0.139**</td>
<td>-0.164***</td>
<td>-0.217***</td>
<td></td>
</tr>
<tr>
<td>(log) rel distance (sea/air)</td>
<td>(0.076)</td>
<td>(0.054)</td>
<td>(0.042)</td>
<td>(0.064)</td>
<td></td>
</tr>
</tbody>
</table>

Observations: 83,460 83,460 80,620 80,608 39,988  
Exporter-product-district FE: Yes Yes Yes Yes Yes  
Product-transport FE: No No Yes Yes No  
District-transport FE: No No No Yes No  
Exporter-transport FE: No No No Yes No  
Exporter–product-transport FE: No No No No Yes  

Note. The dependent variable in all columns is the (log) unit value at the exporter-product-district-transport level. The variable Air Trans equals 1 when the product is transported by air, and it equals 0 when is done by sea. (log) Distance Diff is the log-ratio between sea and air distances, between the exporter and district of entry. Robust standard errors clustered at exporter-district-transport level. *p<0.1, **p<0.05, ***p<0.01.

Transported by sea. In columns (4)-(6), the same is done for the case of airborne imports. The baseline cases in columns (1) and (4) show that, when looking at a given exporter and a given product, those exporters tend to send higher-quality varieties by each mode of transport to more distant US districts given the type of route (sea or air) chosen. Columns (2) and (5) show that those results are robust to controlling for district fixed effects – this would account (among other things) for the possible effect of income differences across regions on the average quality of imports in the presence of nonhomothetic preferences. In addition, columns (3) and (6) include a complete set of district-product fixed effects, in case there exist heterogeneities in preferences for quality of different products across US districts.

Disaggregating exporter-by-product imports by the district of entry and mode of transport can also shed light on the selection of mode of transport along the quality dimension. To that end, column (1) in Table 3 regresses the (log) unit values of imports (regardless of their mode of transport) on the dummy variable Air Trans, which equals 1 when air transportation is used. The regression also includes a full set of PED fixed effects. The result shows that, when comparing the average price of airborne vs seaborne imports of a given product sourced from a given exporter by a given district, airborne imports turn out to be almost 90% more expensive than those shipped by sea. If we consider
unit values as a proxy of product quality, we can interpret this result as evidence that air freight tends to be more prominently used for higher-quality varieties of products.

We can also exploit the fact that relative sea vs air distances to exporters vary quite substantively across different US districts of entry. Such variation allows us to investigate whether the mode of transport selection along the quality dimension responds somehow on the extensive margin to variations in relative sea vs air distances. Columns (2)-(5) in Table 3 approach this question. This set of regressions includes an interaction term $\text{Air Trans} \times (\log) \text{Distance Diff}$, where the variable $(\log) \text{Distance Diff}$ is defined as the logarithm of the ratio sea distance/air distance between the exporter and US district of entry. Column (2) repeats the regression in column (1), adding this interaction term. The coefficient associated with the interaction term is negative and statistically significant. This finding means that the difference in the average price of airborne and seaborne imports tends to narrow as the relative sea vs air distance increases. The following three columns in Table 3 show that this last result is robust to including several other sets of fixed effects.\footnote{For example, in column (3), product-transport fixed effects would control for the possibility that certain products tend to use more intensively one specific mode of transport. In column (4), district-transport fixed effects will account for different intensities of the mode of transport across US districts. Similarly, exporter-transport fixed effects will account for the variation of the mode of transport by country of origin of imports. Finally, the sets of fixed effects in column (5) allow the intensity of mode of transport to vary heterogeneously by district of entry, exporter, and product.}

Considering unit values as a proxy for quality, the results in Table 3 showcase two interrelated empirical observations. When both means of transport are simultaneously used at the PED level:

1. Varieties of the product shipped by air tend to be of higher quality than those shipped by sea.
2. The average quality gap of the varieties shipped by air relative to those shipped by sea narrows as the relative bilateral distance sea vs air increases.

In the next section, we present a model that will intend to rationalise the above two observations as a result of shifts in the extensive margin along which vertically differentiated products choose their mode of transport optimally. The model we propose will lead to selecting the mode of transport along the quality dimension: firms producing higher-quality varieties will use air freight as a mode of transport. However, this selection will also respond to differences in relative bilateral distances between air and sea routes. In particular, consistent with the evidence in Table 1, higher relative distances sea vs air will lead to lower use of sea relative to air freight. This result will materialise as a shift on the extensive margin for the mode of transport selection, leading in turn to a reduction of the average gap in quality of airborne imports relative to seaborne imports.
3 Setup of the Model

There are two types of consumption goods: a single homogeneous good and a large set of varieties of a differentiated good. Goods are non-perishable and can be stored for any required amount of time at zero cost. The world economy comprises a set \( \mathcal{X} \) of countries with producers of the differentiated good and a set \( \mathcal{M} \) of destination countries.

We let each variety of the differentiated good be uniquely identified by the duple \((x, Q)\) \(\in \mathcal{X} \times \{L, H\}\). The index \(x \in \mathcal{X}\) denotes the country where the producer of the variety \((x, Q)\) is located. The index \(Q \in \{L, H\}\) indicates the version of the differentiated good, which reflects the good’s quality level. Quality is denoted by \(\lambda_Q\), with \(0 < \lambda_L < \lambda_H\). For simplicity, we will assume that in each country \(x \in \mathcal{X}\) there exists one producer specialised in the high-quality version \(H\) and one producer specialised in the low-quality version \(L\). Without any loss of generality, we assume henceforth that the homogeneous good is fully sourced domestically.

Time is continuous over the time horizon \(t \in [0, T]\).

3.1 Endowment and Preferences

Consider a generic destination market \(m \in \mathcal{M}\). The demand side of the market comprises a unit mass of individuals indexed by \(i \in \mathcal{I}\). Individuals are born at \(t = 0\) and die at \(t = T\). We assume individuals only consume at the last instant of their lives (i.e., at \(t = T\)). All individuals are born with an identical endowment \(Y_i; 0 = Y_0 > 0\). We assume the size of the endowment changes over time following a bounded random process, identical for all individuals (thus, \(Y_{i,t} = Y_t, \forall i, t \in \mathcal{I} \times [0, T]\)), with \(E_0[Y_i] = Y_0\) and \(\text{Var}_0[Y_i] = \sigma_Y t^{10}\).

Each consumer \(i\) has identical preferences represented by the following indirect PIGL utility function:

\[
V_i = V = -\frac{1}{\varepsilon} \left[ \frac{1}{Y} \left( \sum_{x \in \mathcal{X}} \lambda_L p_{x,L}^{1-\sigma} \right)^{\frac{\eta}{1-\sigma}} p_b^{1-\eta} \right]^{\frac{\varepsilon}{\eta}} - \frac{1}{p_b} \left( \sum_{x \in \mathcal{X}} \lambda_H p_{x,H}^{1-\sigma} \right)^{\frac{1}{1-\sigma}},
\]

where \(p_b\) is the price of the homogeneous good (indexed by the letter \(b\)), \(p_{x,Q}\) is the price of variety \((x, Q)\), and \(Y\) is the size of the endowment at the time goods are purchased\(\text{\footnote{1\textsuperscript{11}}}}\). Regarding the preference parameters, we impose the following restrictions: \(\varepsilon \in (0, 1)\), \(\eta \in (0, 1/2]\), and \(\sigma > 1\).\footnote{1\textsuperscript{12}}
The utility function (1) can be re-stated more compactly by letting \( P_Q \equiv \left( \sum_{x \in X} \lambda Q x^1_{x,Q} \right)^{1/(1-\sigma)} \) denote the price index of a composite good comprising all versions \( Q \in \{L, H\} \) of the differentiated goods, wherever produced. In this way we can write:

\[
V = -\frac{1}{\varepsilon} \left( \frac{P_L^{\eta} p_b^{-\eta}}{Y} \right)^{\varepsilon} - \frac{P_H}{p_b},
\]

### 3.2 Demand Functions

Applying Roy’s identity to the indirect utility function (1) gives the Marshallian demand functions:

\[
q_b = \frac{1}{p_b} \left[ (1 - \eta) Y - \frac{P_L^{\eta \varepsilon} P_H}{p_b^{1+(1-\eta)\varepsilon}} Y^{1+\varepsilon} \right],
\]

\[
q_{x,L} = \lambda_L (\eta P_L^{\sigma-1}) \frac{P_L^{-\eta \varepsilon} P_H}{p_b^{1+(1-\eta)\varepsilon}} Y^{1+\varepsilon},
\]

\[
q_{x,H} = \lambda_H \frac{P_L^{-\eta \varepsilon} P_H}{p_b^{1+(1-\eta)\varepsilon}} Y^{1+\varepsilon}.
\]

As it is typical when dealing with PIGL indirect utility functions, a restriction must be imposed for the the standard properties of a demand function to hold. In this case, the restriction reads:

\[
Y < \bar{Y} \equiv (1 - \eta)^{1/\varepsilon} p_b^{1-\eta+1/\varepsilon} P_L^{\eta} P_H^{-1/\varepsilon},
\]

and it ensures that the demand for the homogeneous good is always positive. (Instead, the demand for any differentiated good is positive as long as \( Y > 0 \).)

Demand for the low-quality version of the differentiated good has a unit expenditure elasticity, i.e. \( \epsilon_{x,L} = (Y/q_{x,L}) (\partial q_{x,L}/\partial Y) = 1 \), whereas the demand for the high-quality version is expenditure elastic, i.e. \( \epsilon_{x,H} = 1 + \varepsilon > 1 \). As a result, high-quality goods are luxuries, while low-quality goods are not.

The expenditure elasticity of demand for the homogeneous good is:

\[
\epsilon_b = 1 - \frac{\varepsilon \left( \frac{P_L}{p_b} \right)^{-\eta \varepsilon} \left( \frac{Y}{p_b} \right)^{1+\varepsilon}}{(1 - \eta) \frac{Y}{p_b} \left( \frac{P_L}{p_b} \right)^{-\eta \varepsilon} \left( \frac{Y}{p_b} \right)^{1+\varepsilon}} < 1.
\]

Unlike the differentiated good, the homogeneous good’s expenditure elasticity is not constant. Specifically, it declines with the expenditure level, i.e. \( \partial \epsilon_b / \partial Y = - (1 - \epsilon_b) (\varepsilon + 1 - \epsilon_b) / Y < 0 \). As the size of the expenditure rises, the homogeneous good goes from being considered a necessity to an inferior good. The switch occurs at the expenditure level \( \bar{Y} = (1 + \varepsilon)^{-1/\varepsilon} \bar{Y} \).

---

\(^{13}\) For sake of completeness, note that \( \lim_{Y \to 0} \epsilon_b = 1 \) and \( \lim_{Y \to \bar{Y}} \epsilon_b = -\infty \).

3.3 Geography and Transport Costs

Now, consider a generic country of origin $x \in \mathcal{X}$. From country $x$’s viewpoint, destination markets differ in terms of their geographic distance. We let $m$ be the distance between countries $x$ and $m$, and assume sea shipping takes one unit of time per each unit of distance. Instead, air shipping from $x$ to any $m$ is assumed to be instantaneous, regardless of the value of $m$.

Exporters choose the exact moment $t$ during $[0, T]$ when to place their goods at the exact port of departure they will use (that is, either their local seaport or airport). Note that if a destination market were located farther from $x$ than $T$, (i.e., $m > T$), then producers would be unable to sea ship their goods.

Producers incur an iceberg cost when shipping goods to each $m$. For seaborne exports, the cost is proportional to the distance $m$. Specifically, the iceberg transport cost is $\tau S m$, where $\tau S$ is the sea shipping cost per unit of distance. For airborne exports, the cost is specific to the country of origin, but it is the same for all destination and denoted $\tau A$. We assume that $0 < \tau S m < \tau A$.

Profits-maximising producers face a constant marginal cost $c_Q$ along with the transportation cost. As the next section illustrates, the timing of producers’ choices is essential in understanding their profit prospects and eventually their decisions on how to ship their products.

4 Choice of Mode of Transport

Producers may opt for air or sea shipping their goods. Recall that sea shipping is cheaper but more time consuming than air shipping, and that the variance of the distribution of time-$T$ expenditure is higher the earlier the shipment has to depart in order to reach its destination at time $T$. Jointly considered, these two features generate a trade-off between the two transport modes, which the producer resolves by comparing the relevant expected profits. In this section, we consider a generic producer of version $Q \in \{L, H\}$ in country $x \in \mathcal{X}$ and look separately into the profit prospects that would obtain by opting for air and sea shipping. We then contrast the two resulting expressions to identify and analyse the condition for choosing between transport modes.

The assumption regarding consumption taking place only at $t = T$ has important implications regarding the timing of shipping. A producer choosing sea transport mode will optimally choose the departure time at time $T - m$. This is so because $\text{Var}_{T-m}[Y_T]$ is the minimised value of expenditure volatility compatible with sea shipping from $x$ to $m$. Any seaborne shipment departing at $t < T - m$ is suboptimal (as it will carry higher variance than $\text{Var}_{T-m}[Y_T]$). Any seaborne shipment departing at $t > T - m$ would not deliver the goods on time. Likewise, a producer choosing air transport mode will optimally opt for the departure time $t = T$, since air shipping is instantaneous.

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\[14\] We abstract from referring to country $x$ whenever possible to ease notation.

\[15\] A destination market such that $m = 0$ can be interpreted as local (or domestic) market.
### 4.1 Sea-Transport Profit

When opting for sea shipping, the market price will be determined at time $T$, whereas the producer must decide on the quantity to deliver at $T - m$, the seaborne shipment optimal departure time. Letting $q_Q$ be the pre-determined shipping quantity and taking into account the relevant expression between (4) and (4), at time $T$ the producer will set the price:

$$p_Q = \left(\frac{\Omega_Q}{q_Q}\right)^{\frac{1}{\sigma}} Y^{\frac{1+\varepsilon_Q}{\sigma}}.$$  \hfill (6)

The producer will then obtain the (ex-post) profit:

$$\pi_Q^S = \left[\left(\frac{\Omega_Q}{q_Q}\right)^{\frac{1}{\sigma}} Y^{\frac{1+\varepsilon_Q}{\sigma}} - (1 + \tau_{SM}) c_Q\right] q_Q,$$  \hfill (7)

where $\tau_{SM}$ is the iceberg transport cost incurred by the producer, along with the constant marginal cost of production $c_Q$, $Q \in \{L, H\}$, with $\varepsilon_L = 0$, $\varepsilon_H = \varepsilon$, and:

$$\Omega_Q \equiv \lambda_Q \left(\frac{P^\sigma}{P_L^\sigma - \frac{P_H^\sigma}{1+(1-\eta)\varepsilon}}\right)^{\varepsilon_Q/\varepsilon}.$$  \hfill (8)

At time $T - m$, the producer forms expectations on profit (7) and uses the resulting expression to pinpoint the optimal quantity to sea ship, obtaining:

$$q_Q = \left(\frac{\sigma - 1}{\sigma}\right)^{\frac{1}{\sigma}} \frac{\Omega_Q}{(1 + \tau_{SM}) c_Q} E_{T - m} \left[Y_T^{\frac{1+\varepsilon_Q}{\sigma}}\right]^\sigma.$$  \hfill (9)

Plugging this expression into (7) and taking expectations at time $T - m$ yields:

$$E_{T - m} \left[\pi_Q^S\right] = \left(\frac{\sigma - 1}{\sigma}\right)^{\frac{1}{\sigma}} \frac{\Omega_Q}{(1 + \tau_{SM}) c_Q} E_{T - m} \left[Y_T^{\frac{1+\varepsilon_Q}{\sigma}}\right]^\sigma.$$  \hfill (10)

### 4.2 Air-Transport Profit

When choosing air shipping, the producer incurs the iceberg transport cost $\tau_A$ (along with the constant marginal cost of production $c_Q$). The shipment will depart at time $T$. The producer, taking the relevant demand into account, either (4) or (5), will solve the profit maximisation problem:

$$\pi_Q^A = \max_p [p - (1 + \tau_A) c_Q] \Omega_Q p^{-\sigma} Y_T^{1+\varepsilon_Q}.$$  

The solution of the problem delivers the optimal price that the producer will set at time $T$:

$$p_Q = \frac{\sigma}{\sigma - 1} (1 + \tau_A) c_Q,$$  \hfill (11)

leading to the (ex-post) profits:

$$\pi_Q^A = \frac{(\sigma - 1)^{\frac{1}{\sigma}}}{\sigma} \frac{\Omega_Q Y_T^{1+\varepsilon_Q}}{[(1 + \tau_A) c_Q]^{\sigma-1}}.$$  \hfill (12)
As we noted above, the producer optimally takes the transport mode decision at time $T_m$. The relevant object to comparing profit opportunities is then the expected value, taken at time $T_m$, of the ex-post profits $\pi_{Q}$, which yields:

$$E_{T-m}[\pi_{Q}] = \frac{(\sigma - 1)^{\sigma-1}}{\sigma} \frac{\Omega Q}{(1 + \tau A) c_{Q}}^{\sigma-1} E_{T-m} \left[ Y_{T}^{1+\varepsilon_Q} \right].$$

### 4.3 Profit Comparison (Air vs Sea)

The producer takes the decision on the transport mode by comparing the expected profits (13) and (10), for air and sea shipping, respectively. Specifically, the producer chooses air transportation only if $E_{T-m}[\pi_{Q}^A] \geq E_{T-m}[\pi_{Q}^S]$, which leads to the following result.

**Lemma 1** Inequality $E_{T-m}[\pi_{Q}^A] \geq E_{T-m}[\pi_{Q}^S]$ yields the condition:

$$\Phi_{m,Q} = \left\{ \frac{2\sigma Y_{T-m}^2 + \sigma \varepsilon_Q Var_{T-m}[Y_T]}{2\sigma Y_{T-m}^2 + [\varepsilon_Q - (\sigma - 1)] Var_{T-m}[Y_T]} \right\}^{1+\varepsilon_Q} \left( \frac{1 + \tau S m}{1 + \tau A} \right)^{\sigma-1} \geq 1. \quad (14)$$

**Proof.** See Appendix. ■

We may now study some key comparative statics exercises regarding the producer’s transportation mode choice.

**Proposition 1 (Comparative Statics)** Condition (14) is less restrictive:

1. for higher-quality goods, formally: $\Phi_{m,H} > \Phi_{m,L}$;
2. if the expenditure is more volatile: (2.i) $\partial \Phi_{m,Q}/\partial Var_{T-m}[Y_T] > 0$; and more so for higher-quality goods: (2.ii) $\partial \Phi_{m,H}/\partial Var_{T-m}[Y_T] > \partial \Phi_{m,L}/\partial Var_{T-m}[Y_T]$;
3. if the distance to the destination market increases: $\partial \Phi_{m,Q}/\partial m > 0$.

**Proof.** See Appendix. ■

The results in Proposition 1 represent a collection of events whose occurrence would induce producers to choose airborne shipping. Except for the last result, all findings directly or indirectly involve expenditure volatility. The reason is that the core mechanism of our model hinges on the risk of choosing the quantity to ship to the destination market in advance. For a given producer, supplying goods to a market with more erratic expenditure entails an incentive to opt for air shipping, in order to avoid the constraint of fixing quantity in advance implied by the sea transport mode. Likewise, a more distant destination market makes ocean cargoes less desirable as it requires longer sea shipping time and, furthermore, implies higher seaborne transportation cost.
It is important to understand the role of product quality in shaping the decision on the transport mode. Higher-quality goods are associated with more income-elastic demands. Therefore, the effect of any expenditure variation on the producer’s expected profit is magnified. For this reason, higher-quality goods are relatively more exposed to the risks generated by expenditure volatility. As a result, producers of higher-quality goods are more likely to opt for air shipping than producers of lower-quality goods.

4.4 Discussion

By means of the model’s predictions stated by Proposition 1 and a few thought experiments, we can offer some rationale to the observations illustrated in Section 2. To this purpose, we may interpret how restrictive condition (14) is as indicating how many producers will ship their products by sea: specifically, the higher this degree, the larger the share of seaborne imports. Furthermore, we may consider the distance \( m \) between the country of origin and the destination market as a relative distance. Since \( m \) affects neither the transportation cost nor the profits for airborne imported products, the model implicitly regards the air distance as common to all destination markets.

In light of these considerations, Result (2) of Proposition 1 rationalises the results reported in Table 1. A larger relative sea distance would be captured by a higher value of \( m \), which would induce a raise in \( \text{Var}_{T-m} [Y_T] \) and, thereby, an increase in \( \Phi_{m,Q} \). In other words, by expanding the time elapsing from the moment the producer choose the transportation mode to the moment the products reach their destination market, a larger relative sea distance exacerbates the risk generated by expenditure volatility, making it more likely for producers to opt for air shipping their goods. Note also that \( \partial \Gamma_m / \partial m < 0 \) would reinforce said incentive by magnifying the relative sea (to air) transportation cost.

We may read Result (3) of Proposition 1 as sorting optimal transport choices across producers from the same country of origin supplying goods of different quality levels to the same destination market, with any other condition common to all those producers. In this sense, \( \partial \Phi_{m,Q} / \partial \varepsilon_Q > 0 \) would mean that the producers of higher-quality goods would be more likely to ship by air. If we reverted the logic, this statement would imply that the average quality of airborne imports is higher than that of seaborne imports. Our findings in Columns (1) and (2) of Table 3 show that an analogous regularity is observed in the data.

Furthermore, jointly considering Results (2) and (3) of Proposition 1 help in rationalising the findings in the remaining columns of Table 3. A larger relative sea distance induces additional producers to switch from sea to air shipping. Since those producers invariably manufacture goods of lower quality than those already airborne imported, the average quality of a given product air-shipped from a given country to a given US district declines with rising relative sea distance (sea/air).
5 Conclusion

We developed a model featuring several goods, importers with PIGL preferences and exporters supplying two quality-differentiated versions of each good. Demand uncertainty dictated that the transport mode choice be delayed as far as possible in time. Therefore, profit maximising exporters made their choice comparing the profit expectations resulting from air vs sea shipping their goods, taken at the moment that minimises the time lag of seaborne imports. In line with several observations concerning the relations between import shares, product quality and relative air-to-sea distance, we find that more considerable demand volatility induces more exporters to opt for air shipping, and more so if they produce high-quality goods.

A Appendix

A.1 Auxiliary results and proofs

Proof of Lemma 1. Preliminarily, notice that \( E_{T-m} \left[ \pi_Q^A \right] \geq E_{T-m} \left[ \pi_Q^S \right] \), in conjunction with (13) and (10), yields:

\[
E_{T-m} \left[ Y_T^{1+\varepsilon_Q} \right] \leq \frac{E_{T-m} \left[ Y_T^{1+\varepsilon_Q} \right]^\sigma}{\left[ (1+\tau_A) \right]^{\sigma-1}}.
\]

(15)

Since the stochastic process governing the \( Y \)-dynamics is only defined implicitly, we need to approximate the expectations in (14). Using the Arrow-Pratt certainty equivalent approximation, we could write the expectation of any power function \( Y^\alpha \) as:

\[
E_{T-m} \left[ Y_T^\alpha \right] \approx Y_{T-m}^\alpha \left\{ 1 + \frac{(\alpha - 1) \text{Var}_{T-m} [Y_T]}{2 Y_{T-m}^2} \right\}^\alpha,
\]

(16)

from which we may obtain, respectively setting \( \alpha = 1 + \varepsilon_Q \) and \( \alpha = (1 + \varepsilon_Q) / \sigma \):

\[
E_{T-m} \left[ Y_T^{1+\varepsilon_Q} \right] \approx Y_{T-m}^{1+\varepsilon_Q} \left( 1 + \frac{\varepsilon_Q \text{Var}_{T-m} [Y_T]}{2 Y_{T-m}^2} \right)^{1+\varepsilon_Q},
\]

\[
E_{T-m} \left[ Y_T^{(1+\varepsilon_Q)/\sigma} \right] \approx Y_{T-m}^{1+\varepsilon_Q} \left\{ 1 + \frac{[\varepsilon_Q - (\sigma - 1)] \text{Var}_{T-m} [Y_T]}{2 \sigma Y_{T-m}^2} \right\}^{1+\varepsilon_Q}.
\]

Plugging these expression into (15), and rearranging, (14) obtains.

Proof of Proposition 1.

(1) Preliminarily, notice that \( \partial \ln \Phi_{m,Q} / \partial \varepsilon_Q > 0 \) implies \( \Phi_{m,H} > \Phi_{m,L} \), since by definition \( \varepsilon_H > \varepsilon_L \).

Log-differentiating the left-hand side of condition (14) with respect to \( \varepsilon_Q \) yields:

\[
\frac{\partial \ln \Phi_{m,Q}}{\partial \varepsilon_Q} = \ln \Phi_{m,Q} \frac{1}{1 + \varepsilon_Q} + (1 + \varepsilon_Q) \left\{ \frac{\text{Var}_{T-m} [Y_T]}{2 Y_{T-m}^2} - \frac{\varepsilon_Q \text{Var}_{T-m} [Y_T]}{2 \sigma Y_{T-m}^2} \right\}
\]

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Simplifying and rearranging, we have:

\[
\frac{\partial \ln \Phi_{m,Q}}{\partial \varepsilon_Q} = \frac{\ln \Phi_{m,Q}}{1 + \varepsilon_Q} + \frac{(1 + \varepsilon_Q) \text{Var}_{T-m}[Y_T]}{2\sigma Y_{T-m}^2} \left\{ \frac{\sigma}{1 + \frac{\varepsilon_Q \text{Var}_{T-m}[Y_T]}{2\sigma Y_{T-m}^2}} - \frac{1}{1 + \frac{\varepsilon_Q - (\sigma-1) \text{Var}_{T-m}[Y_T]}{2\sigma Y_{T-m}^2}} \right\}.
\]

First, note that \( \Phi_{m,Q} \geq 1 \). To see this, let \( Z_T \equiv Y_T^{1+\varepsilon_Q} \). We have \( \Phi_{m,Q} = E_{T-m} \left[ Z_T^{1/\sigma} \right] \), which by applying the Arrow-Pratt certainty equivalent approximation yields \( \Phi_{m,Q} \approx \left\{ 1 - (\sigma - 1) \left( 2\sigma Y_{T-m}^2 \right)^{-1} \text{Var}_{T-m}[Y_T] \right\}^{-1} > 1 \), since \( \sigma > 1 \). Second, suppose that \( 2\sigma Y_{T-m}^2 \leq \text{Var}_{T-m}[Y_T] \). Then, for any \( \alpha < 0 \), \( 2\sigma Y_{T-m}^2 - (\sigma - 1) \text{Var}_{T-m}[Y_T] < 0 \). But if this inequality were satisfied, then \( [16] \) would imply \( E_{T-m} \left[ Y_T^\alpha \right] < 0 \) for \( \alpha < 0 \), which cannot possibly hold since expenditures are bounded away from zero. As a consequence, it must be that \( 2\sigma Y_{T-m}^2 > \text{Var}_{T-m}[Y_T] \), which in conjunction with \( \Phi_{m,Q} \geq 1 \) in turn implies that \( \partial \ln \Phi_{m,Q}/\partial \varepsilon_Q > 0 \). Then, it must be that \( \Phi_{m,H} > \Phi_{m,L} \).

(2.i) Log-differentiating the left-hand side of condition \([14]\) with respect to \( \text{Var}_{T-m}[Y_T] \) yields:

\[
\frac{\partial \ln \Phi_{m,Q}}{\partial \text{Var}_{T-m}[Y_T]} = (1 + \varepsilon_Q) \left\{ \frac{\varepsilon_Q}{2\sigma Y_{T-m}^2 + \varepsilon_Q \text{Var}_{T-m}[Y_T]} - \frac{\varepsilon_Q - (\sigma - 1)}{2\sigma Y_{T-m}^2 + \left[ \varepsilon_Q - (\sigma - 1) \text{Var}_{T-m}[Y_T] \right]} \right\}.
\]

Simplifying and rearranging, we have:

\[
\frac{\partial \ln \Phi_{m,Q}}{\partial \text{Var}_{T-m}[Y_T]} = \frac{(\sigma - 1)(1 + \varepsilon_Q)^2}{\sigma \left( 1 + \frac{\varepsilon_Q \text{Var}_{T-m}[Y_T]}{2\sigma Y_{T-m}^2} \right) \left\{ 1 + \frac{\varepsilon_Q - (\sigma - 1) \text{Var}_{T-m}[Y_T]}{2\sigma Y_{T-m}^2} \right\}} > 0. \tag{17}
\]

That is, a larger expenditure volatility raises the value of \( \Phi_{m,Q} \), thereby mitigating condition \([14]\).

(2.ii) Working out \([17]\) for \( Q = L \) and \( Q = H \), we respectively obtain:

\[
\frac{\partial \ln \Phi_{m,L}}{\partial \text{Var}_{T-m}[Y_T]} = \frac{\sigma - 1}{\sigma} \left\{ 1 - \frac{\varepsilon_Q - (\sigma - 1) \text{Var}_{T-m}[Y_T]}{2\sigma Y_{T-m}^2} \right\},
\]

and:

\[
\frac{\partial \ln \Phi_{m,H}}{\partial \text{Var}_{T-m}[Y_T]} = \frac{\sigma - 1}{\sigma} \left( 1 + \frac{\varepsilon_Q \text{Var}_{T-m}[Y_T]}{2\sigma Y_{T-m}^2} \right) \left\{ 1 + \frac{\varepsilon_Q - (\sigma - 1) \text{Var}_{T-m}[Y_T]}{2\sigma Y_{T-m}^2} \right\}.
\]

The last expression can also be written as:

\[
\frac{\partial \ln \Phi_{m,H}}{\partial \text{Var}_{T-m}[Y_T]} = \Omega_m \cdot \Psi_m \cdot \frac{\partial \ln \Phi_{m,L}}{\partial \text{Var}_{T-m}[Y_T]},
\]

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where:
\[
\Omega_m \equiv \frac{1 + \varepsilon}{1 + \frac{2 \text{Var}_{T-m} [Y^T_m]}{2Y^2_{T-m}}}, \quad \text{and} \quad \Psi_m \equiv \frac{1 + \varepsilon}{1 + \frac{[\varepsilon - (\sigma - 1)] \text{Var}_{T-m} [Y^T_m]}{2\sigma Y^2_{T-m}}}.
\]

Note that \(\Omega_m > 1\) and \(\Psi_m > 1\), since \(\varepsilon > 0\) and \(2Y^2_{T-m} > \text{Var}_{T-m} [Y^T_T]\).

To see this, suppose on the one hand that \(m_1 > 1\) and \(m_2 > 1\), since \(\varepsilon > 0\) and \(2Y^2_{T-m} > \text{Var}_{T-m} [Y^T_m]\). Then, for any \(\alpha < 0\), \(2Y^2_{T-m} - (\alpha - 1) \text{Var}_{T-m} [Y^T_T] < 0\). But if this inequality were satisfied, then (16) would imply \(E_{T-m} [Y^2_{m}] < 0\), which cannot possibly hold for any \(\alpha \in \mathcal{R}\) since expenditures are bounded away from zero.

As a consequence, it must be that \(2Y^2_{T-m} > \text{Var}_{T-m} [Y^T_T]\), which leads to \(\Omega_m > 1\).

On the other hand, suppose that \(\Psi_m < 1\). We would then have:
\[
(1 + \varepsilon) \left\{ \frac{1 - (\sigma - 1) \text{Var}_{T-m} [Y^T_T]}{2\sigma Y^2_{T-m}} \right\} \leq 1 + \frac{[\varepsilon - (\sigma - 1)] \text{Var}_{T-m} [Y^T_T]}{2\sigma Y^2_{T-m}}
\]
which would once again require \(2Y^2_{T-m} \leq \text{Var}_{T-m} [Y^T_T]\). As a consequence, it must hold that \(\Psi_m > 1\), which together with \(\Omega_m > 1\) in turn implies that \(\partial \ln \Phi_{m,H} / \partial \text{Var}_{T-m} [Y^T_T] > \partial \ln \Phi_{m,L} / \partial \text{Var}_{T-m} [Y^T_T]\) as claimed.

(3) Let:
\[
\Delta_{m,Q} \equiv \left\{ \frac{2\sigma Y^2_{T-m} + \sigma \varepsilon Q \text{Var}_{T-m} [Y^T_T]}{2\sigma Y^2_{T-m} + [\varepsilon Q - (\sigma - 1)] \text{Var}_{T-m} [Y^T_T]} \right\}^{1+\varepsilon Q},
\]
\[
\Gamma_m \equiv \left( \frac{1 + \tau sm}{1 + \tau A} \right)^\sigma - 1,
\]
so that (14) reads:
\[
\Phi_{m,Q} = \Delta_{m,Q} \Gamma_m
\]

Log-differentiating this expression with respect to \(m\) yields:
\[
\frac{\partial \ln \Phi_{m,Q}}{\partial m} = \frac{\partial \ln \Delta_{m,Q}}{\partial \text{Var}_{T-m} [Y^T_T]} \frac{\partial \text{Var}_{T-m} [Y^T_T]}{\partial m} + \frac{\partial \ln \Gamma_m}{\partial m}
\]
\[
= \frac{\partial \ln \Phi_{m,Q}}{\partial \text{Var}_{T-m} [Y^T_T]} \xi_Y + (\sigma - 1) \frac{\tau s}{1 + \tau sm} > 0.
\]

QED. ■
References


