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By Force of Confidence

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By force of confidence[*]

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Abstract

Recent macro-finance contributions explain a great deal of unconditional asset pricing by introducing persistent consumption risks and rare disasters. Only the volatility puzzles remain unresolved among the longer-established issues in this literature. Motivated by empirical finance contributions and conventional wisdom, we abstract from a consumption-centric analysis and let the asset-pricing kernel depend on habit formation and consumer confidence as a demand shifter correlated with consumption growth. The resulting model compares favorably with the literature in explaining the risk-free rate volatility, but it falls short in matching the standard deviation of the market return. Our findings justify using supplementary information to price assets while warning against neglecting a thorough analysis of consumption growth dynamics.

JEL Classification: G12, E21

Keywords: Asset Pricing, Consumer Confidence, Habit Persistence, Recursive Utility, Utility from Anticipation, Year-on-Year Growth

1 Introduction

In the last four decades, macro-finance models have gone a long way to explain the concurrent behavior of consumption growth, risk-free rate, equity return, and dividend yield. A number of breakthroughs have recently been achieved by considering preferences for early resolution of uncertainty, persistent consumption risks, and macroeconomic events resulting in rare disasters.[1]

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[1]Seminal works of these branches of the macro-finance literature include Epstein and Zin (1989), Weil (1989), Rietz (1988), Barro (2006), and Bansal and Yaron (2004), which culminated in the contribution owed to Barro and Jin (2021).

A key advantage of these approaches is that predictions obtain from analyzing consumption growth in isolation. However, this feature might also prevent other observables from explaining the origin of risk persistence or the human reaction to distressing events, as suggested elsewhere in the literature.[2] Indeed, casual observation suggests that the conventional focus is not limited to analyzing consumption dynamics; it also encompasses other notions, often of emotional nature.

This paper incorporates observables reflecting consumers' psychological traits into a macro-finance model to produce a persistent state variable correlated with consumption growth. The resulting stochastic discount factor (SDF) appropriately price assets without resorting to persistent risks or rare events. The predictions match those found in the literature concerning the first moments of the market return and risk premium. Importantly, they compare favorably with regard to the volatility of the risk-free rate, providing sensible justification for using additional notions to explain asset pricing. Nevertheless, they also show a limited ability to capture asset prices' variability, suggesting that an exhaustive analysis of consumption dynamics must not be overlooked.

The model considers two prominent aspects that consistently emerge from several contributions in empirical finance as well as macroeconomic, business, and political news. The first aspect concerns information sources. A string of empirical contributions in the financial literature considers confidence indicators' potential role as conditioning information in factor asset pricing models.[3] The financial markets, the media, and the business community hold consumer confidence indicators in high regard when assessing or forecasting economic and financial conditions. Confidence is generally interpreted as an indicator of prospective changes in consumers' income or wealth. Higher confidence, the typical story goes, signals better economic conditions; this induces consumers to feel richer and, accordingly, more prone to consume. We let this conventional wisdom guide our modeling strategy. Consumer confidence plays the role of an exogenous demand shifter, thereby signaling a regime of favorable or critical attitude towards consumption by influencing its marginal utility.[4]

The second aspect is practical: growth rates are computed using the year-on-year convention,

[2] For a disquisition on this matter, see, e.g., Constantinides (2017). Other comprehensive surveys of the macro-finance literature can be found in Mehra (2012), Ludvigson (2013), Campbell (2015), and Cochrane (2017).

[3] Ho and Hung (2009) and Bathia and Bredin (2018) include investor sentiment as conditioning information in factor asset pricing models to study the relevance of the size, value, liquidity, and momentum effects on individual stocks returns. Lemmon and Portniaguina (2006) investigate the time-series relationship between consumer confidence and the returns of small stocks. Chung, Hung, and Yeh (2012) study the potential asymmetry of the predictive power of investor sentiment on stock returns during economic expansions and recessions.

[4] Influential papers corroborate this view (see, e.g., Barsky and Sims, 2012). A more conservative approach would consider consumer confidence merely as conditioning information for the events' probability distribution, thereby limiting its role to measuring consumers' subjective beliefs concerning variations in available resources while abstracting from altering their propensity to consume. As we show below, we also look into this alternate setup and conclude that the relevant predictions do not substantially improve those delivered by the standard asset pricing model.

regardless of the (often higher) frequency characterizing the relevant data. This method is hardly ever used in the asset pricing literature. A rare exception is Jagannathan and Wang (2007), who argue that the "use of calendar year returns avoids the need to explain various well-documented seasonal patterns in stock returns, [...] [and] also attenuates the errors that may arise due to ignoring the effect of habit formation on preferences" (p. 1626). Importantly, this statement points out that our approach takes habit persistence into account since the year-on-year convention causes the Euler equation to comprise several higher-frequency growth factor lags, albeit in the compounded form of yearly growth factors. The rest of the model draws on the framework (hereafter referred to as the "standard" model) proposed by Epstein and Zin (1989) and Weil (1989). As such, our model features consumers' preference over the timing of resolution of uncertainty, too.

The model's core mechanism is analogous to the one exploited by the seminal Lucas (1978) "tree" model. If the asset payoffs covary positively with the consumption growth process, then the resulting negative relationship between asset returns and the SDF drives the expected market premium upwards. The newly introduced elements' effect on the marginal utility of consumption adds another layer of variation to the core mechanism. In the presence of a positive correlation between consumer confidence innovation and consumption growth, the impact of confidence on marginal utility invariably reinforces the income effect generated by the change of future consumption's relative price, in turn implied by the variation of returns' potential realizations across states of nature. This process causes larger SDF deviations from its mean, thereby generating higher equity premia for speculative assets. Furthermore, if the demand shifter is positively autocorrelated, habit persistence's time linkages strengthen this effect. Therefore, the SDF volatility is higher when the model takes consumer confidence and year-on-year growth rates into account. This outcome suggests that the novel source of variability acts as a magnifier of asset prices' response to consumption growth fluctuations. As a result, our approach is suited to replicate the observed financial statistics with a lower consumption growth volatility than the standard asset pricing model requires, in a similar fashion as the recent consumption-centric approaches to macro-finance but through a distinct (though not necessarily incompatible) mechanism.

The modeling strategy is parsimonious. We let the joint stochastic dynamics of consumption growth and consumer confidence innovation follow an autoregressive scheme, allowing the two variables to correlate. We estimate the time series parameters and, to facilitate the numerical solution of the model, we use them to implement Tauchen's (1986) method to approximate the continuous-valued autoregression with a discrete Markov chain. Preferences retain the same three-parameter preference specification as in the standard model. We calibrate these parameters by matching the values of three simulated statistics (namely, the first two moments of the risk-free rate and the mean excess return) with the relevant figures observed in the data.

Our analysis shows that one should consider consumer confidence and the year-on-year convention jointly. The outcomes of the model significantly worsen once we either drop from its specification the consumer confidence as a state variable, or we refrain from using the year-on-year convention to compute growth rates, or both (which corresponds to a version of the standard model, here a special case of our approach). This finding is suggestive of a *persistent* role for consumer confidence in influencing the SDF, with lagged signals concurring with the current one in shaping the asset prices' behavior.

The paper is organized as follows. The remaining of this section reviews the contributions in the literature that are more closely related to our investigation. Section 2 illustrates the consumption-based asset pricing model with preferences augmented with an exogenous state variable; it also shows under which specifications of the state variable the stochastic discount factor comes to depend on year-on-year growth rates in an environment characterized by higher-frequency time intervals. Section 3 describes the data we use for our quantitative exercises, details the procedures we adopt to estimate the stochastic process, and offers an intuition regarding the efficacy of our approach. Section 4 calibrates the preference parameters and discusses our findings. Section 5 concludes. The appendix offers some anecdotal support to the conventional wisdom regarding consumer confidence based on Google search engine queries and contains the most relevant mathematical derivations.

Related literature

The paper relates to several studies that investigate the relationship between consumer confidence and consumption growth. Ludvigson (2004) reports that these studies are motivated by empirical evidence suggesting that consumer confidence predicts consumption growth, over and above other commonly used economic indicators. Acemoglu and Scott (1994) rationalize the observed correlation by positing that consumer confidence variations reflect alterations in the degree of economic uncertainty. As such, these variations might alter precautionary savings motives, owing to changes in the forecast variance of consumption. The authors provide evidence that consumer confidence not only covaries with forecast variance, which suggests a positive link between saving and uncertainty.[5] It also correlates with consumption growth. Building on the latter observation, we show that consumer confidence variations may affect the SDF in the absence of time-varying consumption growth volatility.[6]

[5] In contrast, Ludvigson (2004) finds a negative correlation between confidence and uncertainty in U.S. data and argues that precautionary saving motives would lead to a positive relationship between consumption growth and lagged uncertainty, which would contradict the observed positive correlation between confidence and consumption growth.

[6] Examples in which time preference shocks can be regarded as a way to capture the relationship between fluctuations in market sentiment and volatility of asset prices, see Barberis, Shleifer, and Vishny (1998) and Dumas, Kurshev, and Uppal (2009).

Carroll, Fuhrer and Wilcox (1994) argue that the observed correlation between consumer confidence and consumption growth suggests a potential role for habit formation.[7] As such, our paper also relates to papers that incorporate habit persistence through non-time-separable preferences. Habit can be external (Abel, 1990; Campbell and Cochrane, 1999), merely acting as a reference point, or internal, letting consumers' current marginal utility depend on their own past consumption choice (Constantinides, 1990). Our framework incorporates external habit formation. As already mentioned, our findings indicate that both consumer confidence and habit formation are individually instrumental in obtaining a reasonable account of macro-finance facts. We thus contribute to this literature by providing evidence that the two variables play distinctive roles in explaining asset prices.

More broadly, our paper relates to contributions that enrich the instantaneous utility function with additional arguments governing consumers' time preference. These encompass models that include habit formation as well as models that incorporate utility from anticipation.[8] Campbell and Cochrane (1999, p. 208) eloquently state that habit formation "captures a fundamental feature of psychology: repetition of a stimulus diminishes the perception of the stimulus and responses to it." Utility of anticipation represents the symmetric stance in an intertemporal perspective: the anticipation of a future stimulus alters the perception of current stimuli and responses to them. From this perspective, one may interpret habit formation as a measure of the impact on the current marginal utility of consumption of past events' reminiscence; consumer confidence of the anticipation of *future* conditions. In a seminal paper, Loewenstein (1987) explicitly links anticipation to *internal* factors such as the "pleasurable deferral of a vacation, the speeding up of a dental appointment, the prolonged storage of a bottle of expensive champagne" (p. 666), and defines utility from anticipation as proportional to the future stream of utility from personal consumption, a formalization later borrowed by the few contributions providing asset pricing applications: Caplin and Leahy (2001) investigate the role of anxiety in determining the risk-free rate of return and the equity premium; Kuznitz, Kandel, and Fos (2008) study the effect of anticipatory utility on the mean allocation to stocks. Our approach differs from theirs as it considers *external* factors.

The asset pricing literature contains many contributions that, implicitly or explicitly, incorporate state variables. Indeed, Cochrane (2017) argues that virtually every idea behind macro-finance models can be seen as a generalization of the stochastic discount factor obtained by adding a state variable. Our framework explicitly considers a non-separable utility function in consumption and consumer confidence. Early examples of papers worked out in a similar

[7] The authors claim that the presence of habit formation, which implies that lagged consumption growth has predictive power for current consumption growth, might explain the correlation of lagged confidence with current consumption growth as arising from the correlation of lagged confidence with lagged consumption growth.

[8] For a discussion on the origin and the relevance of anticipatory utility, see Frederick, Loewenstein, and O'Donoghue (2002).

fashion include Eichenbaum, Hansen, and Singleton (1988), Aschauer (1985) and Startz (1989), who let the state variable be leisure, government spending, and the stock of durable goods, respectively.[9] More recently, Piazzesi, Schneider, and Tuzel (2007) introduces housing. In all these papers, the state variable is represented by some *good* other than consumption. Conversely, our approach incorporates traits of psychological nature concerning consumers' time preferences.

Our work relates to models with preferences for early resolution (or recursive utility). At least two fundamental branches of the modern micro-finance literature draw on these models: long-run risks (Bansal and Yaron, 2004; Hansen, Heaton, and Li (2008); Bansal, Kiku, and Yaron, 2012); rare disasters (Rietz, 1988; Barro, 2006, 2009) and persistent-rare disasters (Wachter, 2013; Barro and Jin, 2021). To allow for a more transparent comparison with this literature, we stick to the traditional approach and do not calibrate the consumption growth dynamics to accommodate either feature.

The papers closer in spirit to our approach are Melino and Yang (2003) and Albuquerque, Eichenbaum, Luo and Rebelo (2016). In a framework featuring recursive utility, Melino and Yang (2003) introduce a state variable, letting the preference parameters vary across states. In our paper, instead, all preference parameters hold constant and, as such, are not state-contingent. Albuquerque et al. (2016) is an important example of including an asset demand shifter into an asset pricing model with recursive preferences. These authors reverse-engineer the properties that a time preference shock should have to replicate some observed stylized facts in the macro-finance literature. We complement their work by investigating whether the intertemporal linkages created by incorporating consumer confidence and habit persistence may act as measurable fundamentals for the asset demand shifter. Importantly, our framework lets the demand shifter correlate with consumption. From this viewpoint, one might also interpret our model as allowing for the emergence of animal spirits of Keynesian tradition.

2 The model

This section develops a parsimonious macro-finance model with recursive utility incorporating an exogenous state variable. We begin by describing a simple asset pricing framework with a generic state variable, which is possible because the model's derivations are unaffected by this variable's particular definition (as long as it represents quantities beyond the consumer's control). Next, we illustrate the 'baseline' model comprising demand shifter and year-on-year convention to compute growth rates, which constitutes the main object of analysis in the next section. Then, we show that the framework is sufficiently flexible to encompass three model

[9] While these explorations should in principle enhance the performance of the baseline macro-finance approach, at least as long as the newly introduced variables covary positively with consumption growth and the market return, Campbell, Lo, and MacKinlay (1997, p.326) argue that "none of these extra variables greatly improve the ability of the consumption CAPM to fit the data."

decompositions. Each alternative specification obtains, in turn, by defining the state variable while abstracting from the demand shifter or the year-on-year convention, or both.

Recursive utility and state variable

Consider a consumption-based asset pricing model in which the consumers' preferences are represented by a recursive utility function à la Kreps and Porteus (1978), with the one-period utility non-separable in consumption and an exogenous state variable. Formally, we let the representative consumer's lifetime utility U_t from date t onward be represented by the function

$$U_t = \left[(1-\beta)\left(\kappa_t c_t\right)^{\rho} + \beta \mu_t \left\{U_{t+1}\right\}^{\rho}\right]^{\frac{1}{\rho}}, \tag{1}$$

where c is consumption and κ is the state variable. The term $\mu_t\{\cdot\}$ is a 'certainty equivalent' operator, conditional on information at date t, specified as the nonlinear function of the expected value of future lifetime utility

$$\mu_t \left\{U_{t+1}\right\} = \left[E_t \left\{\left(U_{t+1}\right)^{1-\alpha}\right\}\right]^{\frac{1}{1-\alpha}}. \tag{2}$$

The preference parameters $\beta > 0$ and $0 < \alpha \neq 1$ represent the subjective discount factor and the relative risk aversion coefficient, respectively; $0 \neq \rho < 1$ governs the intertemporal elasticity of substitution $\eta \equiv 1/(1-\rho)$.[10]

Under this preference specification, the stochastic discount factor (SDF) is given by

$$m\left\{s_t, s_{t+1}\right\} = \beta\left(x_{t+1}\right)^{\rho-1}\left(\gamma_{t+1}\right)^{\rho}\left(\frac{V\left\{s_{t+1}\right\}}{\mu_{s_t}\left\{V\left\{s_{t+1}\right\}\right\}}\right)^{1-\rho-\alpha}, \tag{3}$$

where $x_{t+1} \equiv c_{t+1}/c_t$ and $\gamma_{t+1} \equiv \kappa_{t+1}/\kappa_t$ are respectively the consumption and the state variable growth factors, $V\{\cdot\}$ is the value function in equilibrium, and $s = (\kappa, \gamma, c, x)$ denotes the aggregate state.[11]

The SDF incorporates three terms. The first term, $\beta\left(x_{t+1}\right)^{\rho-1}$, is the product between the subjective discount factor and a non-increasing power function of consumption growth. It represents the SDF in the seminal contribution by Mehra and Prescott (1985). The third term, $\left(V\left\{s_{t+1}\right\}/\mu_{s_t}\left\{V\left\{s_{t+1}\right\}\right\}\right)^{1-\rho-\alpha}$, involves the representative consumer's value function and reflects the consumer's preferences for the timing of resolution of uncertainty. Together with the first term, it comprises the SDF in the standard model. If early resolution is preferred,

[10] More precisely, the expression

$$\log U_t = (1-\beta)\log\left\{\kappa_t c_t\right\} + \beta \log\left\{\mu_t\left\{U_{t+1}\right\}\right\}$$

replaces (1) whenever $\rho = 0$.

[11] See Appendix A.2 for a formal derivation of equation (3).

i.e., $1-\rho-\alpha<0$, then asset payoffs in states where realized lifetime utility is lower than the conditional certainty equivalent will have a greater impact on the asset price than payoffs in states where the opposite occurs. Here, however, the value function also depends on the state variable: that is, the latter affects the magnitude of the potential rise in the volatility of the SDF relative to that generated by the first term. The second term, $\left(\gamma_{t+1}\right)^{\rho}$, is a concave function of the innovation in the state variable. Taken in isolation, it reflects the impact of the state variable on the representative consumer's choice abstracting from uncertainty.[12]

Baseline model and decomposition

We may specialize the model by giving the state variable an explicit definition. We generate four different model specifications. We begin with the one representing the baseline model (labeled *CCHF*), which incorporates both consumer confidence and habit persistence. The other three specifications follow from decomposing the baseline model. Specifically, the first specification (*CC*) abstracts from habit persistence; the second (*HF*) abstracts from consumer confidence; the third disregards both elements, in line with the standard model (here labeled *EZW*, as it represents a version of the models in Epstein and Zin, 1989, and Weil, 1989.) Each state variable definition identifies a different SDF, which we will use to perform our quantitative analysis in the next section. To illustrate the link between habit persistence and year-on-year convention in a transparent fashion, it proves convenient to state the length of a model's period explicitly: in line with our quantitative analysis, we let a quarter represent the time elapsing between the dates t and $t+1$.

We begin by simultaneously considering consumer confidence and habit persistence, recreating the specification of the baseline model. Let $\kappa_t^{CCHF} \equiv \psi_t \cdot \varphi_t \cdot \xi_t$, where ψ_t denotes the value of consumer confidence at date t,

$$\varphi_t \equiv \psi_{t-1} \cdot \psi_{t-2} \cdot \psi_{t-3} \tag{4}$$

is a composite function defined over three lagged values of consumer confidence, and

$$\xi_t \equiv \left(c_{t-1} \cdot c_{t-2} \cdot c_{t-3}\right)^{(\rho-1)/\rho} \tag{5}$$

is another composite function defined over three lagged values of consumption. The state variable growth rate becomes

[12] If the consumer is indifferent to the timing of resolution of uncertainty, i.e. $1-\rho-\alpha=0$, then the SDF is ordinally equivalent to $m\left\{s_{t+1}\right\}=\beta\left(x_{t+1}\right)^{-\alpha}\left(\gamma_{t+1}\right)^{1-\alpha}$. In this case, the term $\left(\gamma_{t+1}\right)^{1-\alpha}$ captures the conditional response of consumer choice to the state variable innovation: payoffs in states where innovation is above average have a smaller impact than payoffs in states where the opposite occurs if the coefficient of relative risk aversion is larger than one, and vice versa.

$$\gamma_{t+1}^{CCHF} \equiv \frac{\kappa_{t+1}^{CCHF}}{\kappa_t^{CCHF}} = \frac{\psi_{t+1}}{\psi_{t-3}} \left(\frac{c_t}{c_{t-3}} \right)^{(\rho-1)/\rho} .$$

Denoting $\tilde{\theta}_{t+1} \equiv \psi_{t+1}/\psi_{t-3}$ and $\tilde{x}_{t+1} \equiv c_{t+1}/c_{t-3}$, and using γ_{t+1}^{CCHF} to replace γ_{t+1} into (3), the resulting SDF reads

$$m^{CCHF} \{s_t, s_{t+1}\} = \beta \left(\tilde{x}_{t+1}\right)^{\rho-1} \left(\tilde{\theta}_{t+1}\right)^{\rho} \left(\frac{V^{CCHF} \{s_{t+1}\}}{\mu_{s_t} \{V^{CCHF} \{s_{t+1}\}\}} \right)^{1-\rho-\alpha} , \tag{6}$$

where V^{CCHF} indicates the value function that arises in equilibrium when κ_t^{CCHF} defines the state variable.

The expression in (6) is the price kernel under the fully specified approach. Comparing (6) with (3), we may note that the state variable explicitly incorporates the consumer confidence growth rate into the SDF *and* entails year-on-year growth rate computations.

The definitions of ψ_t, φ_t, and ξ_t ease the decomposition of κ_t^{CCHF} to obtain alternate specifications of the SDF.

If we set $\psi_t = 1$, $\varphi_t = 1$, and $\xi_t = 1$, the definition of state variable reduces to $\kappa_t^{EZW} = 1$, for all dates t, identifying a specification that abstracts from the state variable altogether. It immediately follows that $\gamma_{t+1}^{EZW} = 1$. Substituting this value for γ_{y+1} into (3), the SDF reduces to

$$m^{EZW} \{s_t, s_{t+1}\} = \beta \left(x_{t+1}\right)^{\rho-1} \left(\frac{V^{,EZW} \{s_{t+1}\}}{\mu_{s_t} \{V^{EZW} \{s_{t+1}\}\}} \right)^{1-\rho-\alpha} . \tag{7}$$

Equation (7) corresponds to the SDF of the standard model.

If we set $\psi_t = 1$ and $\varphi_t = 1$ instead, we introduce external habit persistence into the framework while still abstracting from consumer confidence. This setting corresponds to defining the state variable as $\kappa_t^{HF} = \xi_t$. Using (5), and setting $\gamma_{t+1} = \gamma_{t+1}^{HF} \equiv \kappa_{t+1}^{HF}/\kappa_t^{HF} = (c_t/c_{t-3})^{(\rho-1)/\rho}$ into (3), yields

$$m^{HF} \{s_t, s_{t+1}\} = \beta \left(\tilde{x}_{t+1}\right)^{\rho-1} \left(\frac{V^{C} \{s_{t+1}\}}{\mu_{s_t} \{V^{C} \{s_{t+1}\}\}} \right)^{1-\rho-\alpha} \tag{8}$$

Comparing (8) with (7), we may notice the growth rate of consumption is now computed over four quarters (hence, using the year-on-year convention).

Finally, if we set $\varphi_t = 1$ and $\xi_t = 1$, we incorporate consumer confidence and disregard habit persistence. The relevant state variable is $\kappa_t^{CC} = \psi_t$, with $\gamma_{t+1}^{CC} \equiv \psi_{t+1}/\psi_t$. Using this expression in place of γ_{t+1} into (3) and letting $\theta_{t+1} \equiv \psi_{t+1}/\psi_t$, we obtain

$$m^{CC} \{s_t, s_{t+1}\} = \beta \left(x_{t+1}\right)^{\rho-1} \left(\theta_{t+1}\right)^{\rho} \left(\frac{V^{CC} \{s_{t+1}\}}{\mu_{s_t} \{V^{CC} \{s_{t+1}\}\}} \right)^{1-\rho-\alpha} \tag{9}$$

Under this specification, the SDF explicitly features the consumer confidence growth factor as an exogenous state variable.

3 Descriptive analysis

In this section, we begin by illustrating the data that we use to calibrate the Markov chain governing the model's stochastic process and the preference parameters, and to assess the model's predictions. We also stipulate the joint stochastic behavior of consumption and consumer confidence growth rates. We conclude the section by offering, through a graphical representation, an intuition about the model's suitability to replicate the observed financial asset statistics with reasonable preference parameter values.

Data

We need to feed the model data on consumption growth and consumer confidence innovation to obtain predictions regarding the risk-free rate, the market return, and the dividend yield. Naturally, we also need data on the latter variables to create targets for calibrating the model and assessing its performance. We detail our sources in turn.[13] Our database spans from the third quarter of 1967 to the last quarter of 2018, thereby containing 206 observations. Growth factors and gross returns are computed using the year-on-year convention and the more customary (to the macro-finance literature) quarter-on-quarter convention.

The consumption growth time series is calculated using the U.S. Bureau of Economic Analysis data. The United States personal consumption expenditures on non-durable goods and services, expressed in nominal seasonally adjusted annual rates, are deflated using the seasonally adjusted United States personal consumption expenditures 2012 year-base chain-type price index. The resulting monthly figures are converted in per-capita terms using the United States population. We then average the data at a quarterly frequency.

The consumer confidence innovation's time series is calculated using the Conference Board's Consumer Confidence Index (CC) monthly data, retrieved from the Macrobond Financial database.[14] The index is based on a five-question survey, including queries about current and future general market conditions and job availability. Specifically, the questions seek the respondents' appraisal regarding current (i) business conditions and (ii) employment conditions; and the respondents' expectations six months hence regarding (iii) business conditions, (iv) employment conditions, and (v) their total family income. Each question can be given a positive, negative, or neutral answer. The answers' resulting proportions are seasonally adjusted. For each question, the proportion of positive answers is divided by the sum of the proportions of positive

[13] Unless otherwise specified, the time series are sourced at a monthly frequency from the Federal Reserve Economic Data, available at the webpage: `https://fred.stlouisfed.org`.

[14] For further information, visit the webpage: `macrobond.com`.

and negative answers to obtain an indicator, then standardized using the average indicator of the calendar year 1985 to calculate the index level. The overall index value is calculated as the simple monthly average of the five questions' index levels.[15] The index values are then averaged at a quarterly frequency.

The market return time series is derived from the price and dividend time series of the Standard & Poor's 500 composite index, sourced monthly from Shiller's database.[16] The risk-free rate is calculated using the three-month Treasury bill secondary market rate. Treasury bills rates, market prices, and dividends are expressed in real terms through the same price index used to deflate consumption growth data. In order to aggregate the data at a quarterly frequency, dividends are cumulated over the relevant three months; Treasury bills rates are capitalized over the same period. The market price corresponds to the last month's observation of the quarter. The market return is computed as the sum of the current price and dividends divided by the lagged price.

For robustness, we also use the University of Michigan's Consumer Sentiment Index, sourced from the Macrobond Financial database, as an alternative measure of consumer confidence. The index is constructed similarly to the Conference Board's Consumer Confidence Index, although the sample design and the index estimation are substantially different.[17] This indicator is averaged quarterly over the period covered by our database, too.

Figures 1 and 2 illustrate the time paths of the Consumer Confidence Index innovation and the Consumer Sentiment Index innovation, respectively, against consumption growth and the market return. The mean and standard deviation are 1.0064 and 0.114 for consumption growth, 1.021 and 0.026 for Consumer Confidence Index innovation, 1.002 and 0.061 for the Consumer Sentiment Index innovation, 1.076 and 0.064 for market return. We note a marked tendency to pairwise comovement of the variables involved. This remark is also confirmed by the figures reported in Table 1, which reports the pairwise correlation coefficients of the mean factors of consumption growth, the Consumer Confidence Index, the Consumer Sentiment Index, and market return, along with the mean market dividend yield. Specifically, the table offers a comparison between the correlation arising from quarter-to-quarter (Panel A) and year-on-year (Panel B) computations. It might be noticed that all correlations grow in magnitude when moving from the first to the second set of figures. This fact is consistent with the scenario outlined in the introductory section: habit persistence tends to strengthen the positive correlation between consumption growth and the demand shifter when the latter is positively autocorrelated (in the data, the first-lag autocorrelation coefficient is 0.0247 for the Consumer Confidence Index;

[15] Additional details can be found in the Consumer Confidence Survey Technical note, available at the webpage: `conference-board.org/pdf_free/press/TechnicalPDF_4134_1298367128.pdf`.

[16] Shiller's database is available at the webpage: `econ.yale.edu/~shiller/data/ie_data.xls`.

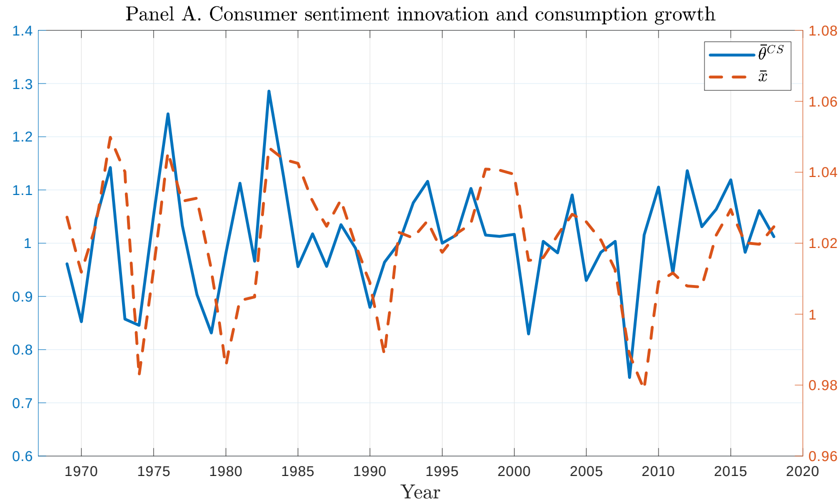
[17] For more information about the Consumer Sentiment Index, visit the webpage: `sca.isr.umich.edu`.

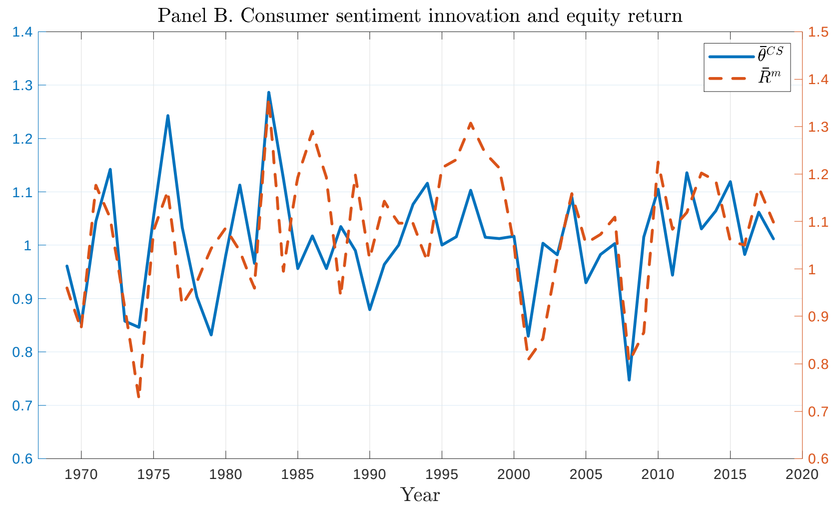
Figure 1.

Consumer confidence, consumption growth and equity return.

Note. The figure illustrates the evolution over time of the Consumer Confidence Index innovation ($\bar{\theta}^{CC}$) against consumption growth ($\bar{x}$, Panel A) and the market return ($\bar{R}^m$, Panel B). The bars over symbols indicate that the data consist of yearly averages of annualized quarterly growth factors (for $\bar{\theta}^{CC}$ and $\bar{x}$) and gross return (for $\bar{R}^m$). The values are pairwise expressed in different scales: the left-hand side refers to $\bar{\theta}^{CC}$, the right-hand side either to $\bar{x}$ (Panel A) or $\bar{R}^m$ (Panel B).

Figure 2.
Consumer sentiment, consumption growth and equity return.





Note. The figure illustrates the evolution over time of the Consumer Sentiment Index innovation ($\bar{\theta}^{CS}$) against consumption growth ($\bar{x}$, Panel A) and the market return ($\bar{R}^m$, Panel B). The bars over symbols indicate that the data consist of yearly averages of annualized quarterly growth factors (for $\bar{\theta}^{CS}$ and $\bar{x}$) and gross return (for $\bar{R}^m$). The values are pairwise expressed in different scales: the left-hand side refers to $\bar{\theta}^{CS}$, the right-hand side either to $\bar{x}$ (Panel A) or $\bar{R}^m$ (Panel B).

Table 1.
Cross correlations.

	(1)	(2)	(3)	(4)
		Panel A. Quarter-on-quarter		
Variable	ω^m_{t+1}	R^m_{t+1}	$\psi^{CS}_{t+1}/\psi^{CS}_t$	$\psi^{CC}_{t+1}/\psi^{CC}_t$
C_{t+1}/C_t	-0.1027	0.1561	0.3294	0.3947
$\psi^{CC}_{t+1}/\psi^{CC}_t$	-0.0844	0.2142	0.6605	
$\psi^{CS}_{t+1}/\psi^{CS}_t$	-0.0436	0.2702		
R^m_{t+1}	-0.0984			
		Panel B. Year-on-year		
Variable	ω^m_{t+4}	R^m_{t+4}	$\psi^{CS}_{t+4}/\psi^{CS}_t$	$\psi^{CC}_{t+4}/\psi^{CC}_t$
C_{t+4}/C_t	-0.1419	0.3760	0.4092	0.5899
$\psi^{CC}_{t+4}/\psi^{CC}_t$	-0.1504	0.4992	0.7859	
$\psi^{CS}_{t+4}/\psi^{CS}_t$	-0.0981	0.5523		
R^m_{t+4}	-0.2519			

Note. The table reports the pairwise cross correlation between consumption growth (C_{t+i}/C_t), the Consumer Confidence Index innovation ($\psi^{CC}_{t+i}/\psi^{CC}_t$), the Consumer Sentiment Index innovation ($\psi^{CS}_{t+i}/\psi^{CS}_t$), the market return ($R^m_{t+i}$), and the market dividend yield (ω^m_{t+i}), using both the quarter-on-quarter ($i = 1$, Panel A) and year-on-year ($i = 4$, Panel B) convention. In Panel A (resp., B), the data consist of annualized quarterly (yearly) growth factors (gross return for R^m_{t+i}).

0.0214 for the Consumer Sentiment Index).

Dynamics of consumption and consumer confidence growth rates

We model the joint process for consumption growth x and consumer confidence innovation θ as the first-order autoregressive scheme

$$\tilde{x}_t = A_{xx}\tilde{x}_{t-1} + A_{x\theta}\tilde{\theta}_{t-1} + \varepsilon_{x,t} \tag{10}$$

$$\tilde{\theta}_t = A_{\theta x}\tilde{x}_{t-1} + A_{\theta\theta}\tilde{\theta}_{t-1} + \varepsilon_{\theta,t} \tag{11}$$

where $\tilde{x}$ and $\tilde{\theta}$ are detrended growth factors, A_{ij}, $i, j = x, \theta$, are autoregression coefficients, and $\varepsilon_{i,t}$ are white noise processes. It is assumed that $\varepsilon_{x,t}$ and $\varepsilon_{\theta,t}$ are mutually independent with cumulative probability $\Pr\{\varepsilon_{it} \le u\} = Z_i\{u/\sigma(\varepsilon_i)\}$, where Z_i is a standardized Gaussian distribution.

Table 2.
Estimated VAR coefficients.

	(1)	(2)	(3)	(4)	(5)	(6)
Coefficient	Baseline CC + HF	CS + HF	CC only	CS only	HF only	Standard
A_{xx}	0.8391	0.8196	0.2665	0.2909	0.8855	0.3225
	(0.0396)	(0.0335)	(0.0713)	(0.0697)	(0.0322)	(0.0661)
$A_{x\theta}$	0.0064	0.0238	0.0322	0.0408		
	(0.0032)	(0.0049)	(0.0162)	(0.0297)		
$A_{\theta x}$	0.5199	−0.7443	0.6625	−0.0287		
	(0.7181)	(0.3493)	(0.3308)	(0.1739)		
$A_{\theta\theta}$	0.7161	0.7819	−0.0349	0.0255		
	(0.0585)	(0.0515)	(0.0753)	(0.0740)		

Note. The table reports the coefficients produced by the VAR(1) estimation of six different model specifications considering: the year-on-year convention and either consumer confidence [column (1)] or consumer sentiment [column (2)] as a demand shifter, or neither [column (5)]; the quarter-on-quarter convention and either consumer confidence [column (3)] or consumer sentiment [column (4)] as a demand shifter, or neither [column (6)]. Two consecutive rows relate to each coefficient: the top one reports the point estimate; the bottom the estimate's standard deviation (in parentheses).

Table 2 reports the estimated coefficients (and their standard deviations) obtained from regressing (10)-(11) using a number of different models and data specifications. Concerning the coefficients of (10), A_{xx} and $A_{x\theta}$, the results are in line with the typical existing evidence across the board: confidence innovation has explanatory power over consumption growth, which is also self-correlated. A distinctive outcome arises instead about (11): the estimated coefficients $A_{\theta\theta}$ are statistically different from zero only when growth factors are computed with the year-on-year convention, regardless of which index (either the Consumer Confidence Index or the Consumer Sentiment Index) proxies the demand shifter in the analysis. This finding suggests that habit formation had interesting novel implications when considered in conjunction with consumer confidence: by letting confidence be more predictable via (11), it improves consumption growth forecasting via (10).

We approximate (10)-(11) with a finite-state Markov chain using Tauchen's (1986) method. The method consists of choosing values of the variables and the transition probabilities for each state so that the resulting discrete Markov chain mimics the underlying continuous-valued autoregression closely. It relies on the well-established Markov chain suitability to adequately capture the relevant time series' statistical properties (after adjusting for trend). The probability of each state is determined by computing the cumulative density for a finite interval of the distributional domain around the values that the two variables take in that particular state. The resulting probabilities comprise the so-called transitional matrix of the Markov chain. By construction, this probability distribution simultaneously accounts for each variable's volatility and autocorrelation, along with the cross-correlation between the two variables.

Consumer confidence, year-on-year growth rates, and the SDF

The different versions of the SDF depicted by (6)-(7) lead to different asset price moments. In order to illustrate why simultaneously incorporating consumer confidence and habit persistence may help in replicating the observed asset pricing behavior, we graphically compare the simulations of the first two moments of the SDF generated by the baseline model (7) and the standard model (6). We then use some basic financial relations to guide our reasoning and develop our intuition.

Figure 3 illustrates the evolution of the mean and standard deviation of the SDF as the relative risk aversion coefficient varies. We set the subjective discount factor to be $\beta = 0.99$ and the intertemporal elasticity of substitution to one ($\rho = 0$). The top panel deals with the expected value of the SDF. We note that the values delivered by the baseline model are, at low levels of RRA, larger than those obtained by the standard framework. The bottom panel concerns the volatility of the SDF. There, the values delivered by the baseline model are substantially higher than the standard framework's at low levels of RRA; the gap narrows as the RRA coefficient rises, yet the magnitude of the SDF volatility generated by the former remains significantly more prominent.

In order to get a quick grasp of how the SDF generates asset prices and the resulting returns, consider the following illustrative exercise. Recall that $cov\left\{m, R\right\} = E\left\{m\, R\right\} - E\left\{m\right\} E\left\{R\right\}$ and $\rho_{m,R} = cov\left\{m, R\right\} / \left(\sigma\left\{m\right\} \sigma\left\{R\right\}\right)$; furthermore, consider that for any asset on the efficient mean-variance frontier it holds that $R \approx a - b\, m$, with a, b some positive numbers, and therefore $\rho_{m,R} = -1$.[18] Then, from the central asset pricing formula, $E\left\{m\, R\right\} = 1$, we may obtain the following three equations that our illustrative simulation must obey

$$E\left\{R^f\right\} = 1/E\left\{m\right\} \tag{12}$$

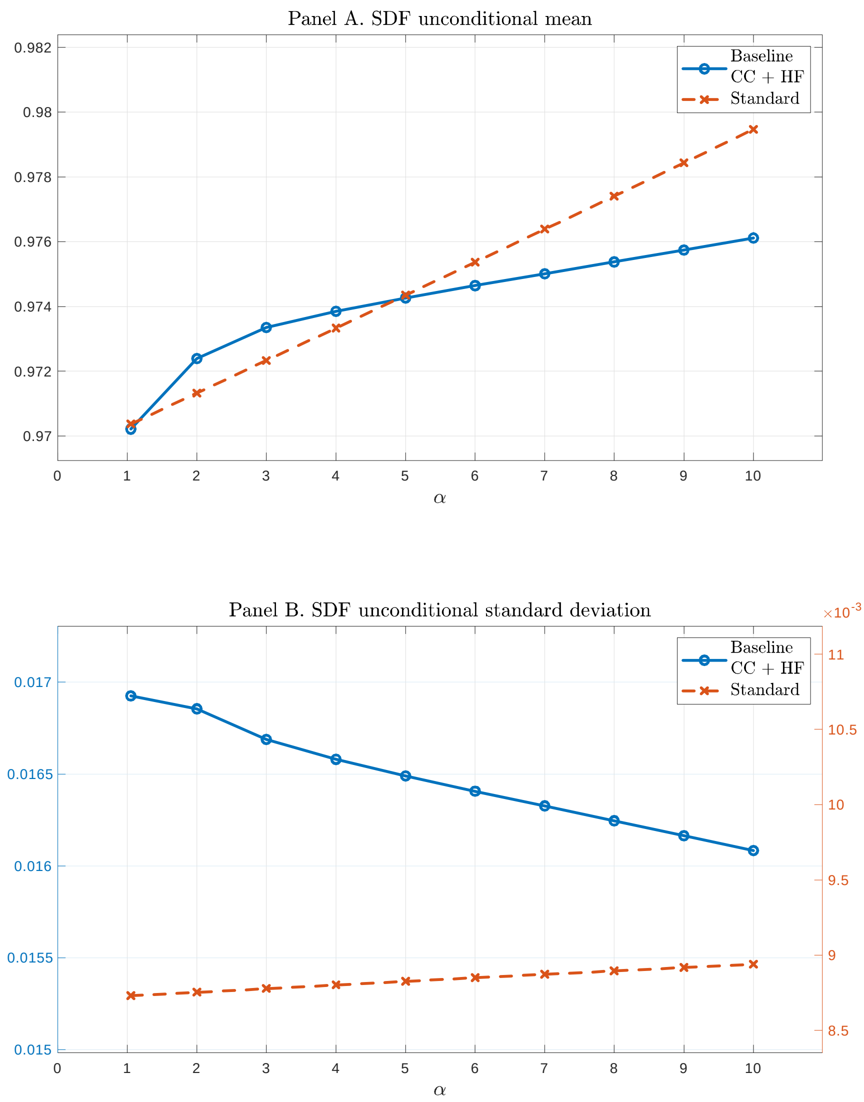
$$E\left\{R^m - R^f\right\} \approx \frac{b\,\sigma^2\left\{m\right\}}{E\left\{m\right\}} \tag{13}$$

$$\sigma\left\{R^m\right\} \approx b\,\sigma\left\{m\right\} \tag{14}$$

[18] More precisely, for $R^m \approx a - b\,m$ to hold, the risky asset should be a good approximation of the market portfolio, and the financial market should not be too far from being complete. For a more exhaustive discussion, see, e.g., Cochrane (2005, Chapter 1).

Figure 3.

SDF average and volatility as the relative risk aversion coefficient varies.



Note. The figure illustrates the patterns of the stochastic discount factor (SDF) mean (Panel A) and standard deviation (Panel B) as the relative risk aversion coefficient varies for two models: the Baseline CC + HF model incorporates both consumer confidence and habit persistence; the Standard model neither. The subjective discount factor is set to $\beta = 0.99$, and the intertemporal elasticity of substitution to one ($\rho = 0$). The SDF standard deviations reported in Panel B are expressed in different scales: the left-hand side refers to the Baseline CC + HF model, the right-hand side to the Standard model.

where $E\left\{R^f\right\}$ and $E\left\{R_m - R^f\right\}$ are the annualized risk-free rate and the market premium unconditional means, $\sigma\left\{R_m\right\}$ is the annualized market return unconditional volatility and b is a value governed by the preference parameters. From (12), we learn that the expected risk-free return is merely the reciprocal of the SDF expected value. Thus, our exercise suggests incorporating consumer confidence and using the year-on-year convention can predict lower riskless rates than a framework abstracting from them for modest levels of risk aversion. From (13), we establish that the market premium is proportional to the ratio between the SDF's variance and mean. In light of our simulation results, we expect model A to predict larger market premia at virtually any level of relative risk aversion. Finally, (14) indicates that the market return volatility is proportional to the SDF's standard deviation. Our simulations are then suggestive of the predictions on $\sigma\left\{R^m\right\}$ following a similar pattern as those on $E\left\{R^m - R^f\right\}$. Each of these three predictions has the potential to represent an improvement over those delivered by the standard model.

4 Quantitative analysis

We now turn to illustrate the model outcomes. We explain the calibration procedure and illustrate and discuss the model predictions. We also critically compare our results with those reported by Barro and Jin (2021) regarding the long-run risks and rare events model as well as those obtained by the standard model proposed by Epstein and Zin (1989) and Weil (1989), here a special case of our approach. We initially focus on the results obtained when considering the Consumer Confidence Index, then we show that our findings are robust to using the Consumer Sentiment Index as a proxy for the demand shifter. Furthermore, we confine our state variable to the role of probability shifter and, finally, produce a robustness check on the transition probability matrix.

Calibration

We need to calibrate two sets of objects to allow the model to deliver the simulated unconditional means and standard deviations of the risk-free rate, the market return, and the dividend yield: the transitional probabilities and the preference parameters governing the consumer's subjective time discounting, relative risk aversion and intertemporal elasticity of substitution. The transitional probability distribution is a prerequisite to running simulations, so we deal with it first. Once the probabilities are calculated, we run an iterative procedure to identify the preference parameters.

As the previous section explains, we use Tauchen's (1986) method to derive the Markov chain probabilities from a continuous-valued stochastic process.[19] The method consists of using

[19] The method is formally discussed in Appendix A.3.

the autoregression coefficients, A_{ij}, $i, j = x, \theta$, and the vector of the error terms, ε_t, obtained by estimating the autoregressive scheme (10)-(11) to compute the variance-covariance matrix of consumption growth and consumer confidence, Σ_y. The elements of Σ_y are then used to produce the values, $\bar{y}_s^v$, that the variables v take in each state s, as well as the relevant Markov chain transitional probabilities, $\pi(s, s')$, from state s to state s'. For each variable, the state-specific values $\bar{y}_s^v$ are equidistant deviations from the variable mean in both directions, with the broader deviation representing the largest shock the variable is allowed to take in the Markov chain. The probabilities associated with the states are the cumulative density of regularly spaced intervals of the joint distributional domain around the values that the two variables take in each given state. The number of states and the magnitudes of the largest shocks must be determined ex-ante. In our exercises, we assign five states ($n = 5$) to each variable (for a total of 25 states jointly) and set the largest shock to be equal to three times ($q = 3$) the magnitude of the relevant standard deviation.

The procedure to determine the three preference parameters is as follows. We search for values of the parameter that minimize a constrained quadratic loss function. The constraints are chosen to reflect the parameter values admitted by the existing contributions in the literature. Specifically, the subjective discount factor can take values no larger than one: i.e., $\beta \in (0.9, 1)$. The relative risk aversion coefficient is assumed to be positive but no larger than 10, representing the upper bound considered reasonable by Mehra and Prescott (1985): i.e., $\alpha \in (0, 10)$. The intertemporal elasticity of substitution (IES) is, as always, lower-bounded in zero. Whether the magnitude of IES may or may not be greater than one is a source of considerable debate.[20] On the one hand, Hall (1988) famously estimates IES to be well below one (around 0.1). On the other hand, a value above one is consistent with the findings of several contributions in the literature since Hansen and Singleton (1982). Furthermore, Bansal and Yaron (2004) show that an above-unity intertemporal elasticity of substitution is essential for rationalizing the observed correlation between consumption volatility and price-dividend ratios. In light of this evidence, we choose $\eta \in (0, 2)$ to constrain the minimization problem concerning the intertemporal elasticity of substitution.

The quadratic loss function is given by the sum of squares of the deviations of the simulated values of three targets (one per parameter) from the observed ones. In our benchmark exercise, the data targets are: (i) the mean of the risk-free rate, $E\left\{R^f\right\}$, to pinpoint the subjective discount factor, β; (ii) the mean of the market return, $E\left\{R^m\right\}$, to pinpoint the relative risk aversion coefficient, α; (iii) the standard deviation of the risk-free rate, $\sigma\left\{R^f\right\}$, to pinpoint the intertemporal elasticity of substitution, η.

[20] For a review of the empirical literature on the intertemporal elasticity of substitution, see Thimme (2017).

Results

We proceed to illustrate the outcomes of the baseline model and its specializations and compare them against the relevant observed statistics. One of the special cases is a version of the standard Epsein-Zin-Weil model, instrumental since it allows for a direct comparison of our results with those produced by previous contributions. We also contrast our findings with those reported by Barro and Jin (2021) concerning the approaches considering long-run risks (LRR), rare events (RE), and their combination (LRR + RE). We subsequently check the robustness of our results along three dimensions. First, we let the Consumer Sentiment Index proxy the demand shifter. Second, we restrain our state variable from acting as a demand shifter. Third, in computing the transition probability matrix, we set to nil the VAR coefficients that are not significantly different from zero.

Table 3 reports the simulated values of a number of statistics, and the calibrated preference parameters used to run the simulations, along with the relevant observed figures. Column (1) details the latter. Columns (2)-(5) correspond to different model specifications. Column (2) gives an account of the outcomes of the baseline model, whose stochastic discount factor (SDF), expressed by (6), includes consumer confidence as a demand shifter in the representative consumer's preference specification, and all moments are computed using the year-on-year convention. In line with our discussion in the previous sections, we refer to the baseline model when simultaneously considering consumer confidence and habit formation. Columns (3)-(5) refer to baseline model's specializations: Column (3) abstracts from habit formation; Column (4) from consumer confidence; Column (5) from both, thereby representing a version of the standard model.

Comparing Column (2) to (1) reveals that the model performs well in replicating the observed average market return and the risk-free rate, and thereby the average risk premium, along with the risk-free rate volatility. Each of these figures represents virtually a 100% match. Furthermore, contrasting Columns (2) and (5) show that our framework roundly outperforms the standard one, which can only account for 19% of the excess return and 34% of the risk-free rate standard deviation. Our results also compare favorably with the ones produced by the LRR and RE approaches (and their combination). While these approaches can also rationalize the risk premium fully, they can only explain up to 30% of the risk-free rate volatility. It is important to note that the baseline model's outcomes obtain with very reasonable calibrated parameter values.

Our approach does not excel in reproducing market return volatility. It only accounts for 10% of the market standard deviation: still better than the standard model's prediction (7%) but worse than the approaches based on LRR/RE (30% to 40%). The reason for this pitfall is the inadequate degree of persistence in expected growth rates generated by habit

Table 3.
Asset pricing statistics: data and predictions.

	(1)	(2)	(3)	(4)	(5)
Statistic	Data	Baseline CC + HF	CC only	HF only	Standard
$E\left\{R^f\right\}$	0.0118	0.0118	0.0340	0.0264	0.0376
$E\left\{R^m\right\}$	0.0758	0.0758	0.0644	0.0569	0.0498
$E\left\{R^m - R^f\right\}$	0.0640	0.0641	0.0304	0.0305	0.0122
$\sigma\left\{R^f\right\}$	0.0229	0.0227	0.0084	0.0066	0.0078
$\sigma\left\{R^m\right\}$	0.1640	0.0156	0.0301	0.0101	0.0122
$\sigma\left\{R^m - R^f\right\}$	0.1618	0.0126	0.0222	0.0036	0.0200
$E\left\{\omega^m\right\}$	0.0291	0.0533	0.0399	0.0344	0.0269
$\sigma\left\{\omega^m\right\}$	0.0121	0.0027	0.0019	0.0032	0.0008
β		0.995	0.999	0.966	0.971
η		1.07	1.99	2.00	1.22
α		5.72	9.49	10.0	9.99

Note. The table reports the observed figures [column (1)] and the simulated values obtained from four different model specifications considering: the year-on-year convention and either consumer confidence [column (2)] or no variable [column (4)] as a demand shifter; the quarter-on-quarter convention and either consumer confidence [column (3)] or no variable [column (5)] as a demand shifter. The variables under scrutiny include the first two moments of the risk-free rate ($E\left\{R^f\right\}$ and $\sigma\left\{R^f\right\}$), market return ($E\left\{R^m\right\}$ and $\sigma\left\{R^m\right\}$), market premium ($E\{R^m{-}R^f\}$ and $\sigma\{R^m{-}R^f\}$), and dividend yield ($E\left\{\omega^m\right\}$ and $\sigma\left\{\omega^m\right\}$). The parameters β, η, and α respectively refer to the subjective discount factor, the intertemporal elasticity of substitution and the relative risk aversion coefficient.

formation relative to that ensured by long-run risks and rare events, from which our approach abstracts. The resulting conditional asset prices vary too little, which causes the model to underestimate market return and dividend yield volatility. The larger SDF standard deviation and the positive covariance with the market return ensure the rise of significant equity risk premia through a low asset price, which also inflates the dividend yield.

It is critical to note that our findings suggest a significant role for both consumer confidence and habit formation. The figures in Columns (3) and (4), which respectively refer to specializations of the model that abstract from habit formation and consumer confidence, mark a general improvement relative to those in Column (5) concerning the standard model. They also signify a marginal improvement over the baseline model regarding the second group of statistics.

Table 4.
Asset pricing statistics: comparison with consumer sentiment predictions.

	(1)	(2)	(3)	(4)	(5)
Statistic	Data	Baseline CC + HF	CC only	CS only	CS + HF
$E\left\{R^f\right\}$	0.0118	0.0118	0.0340	0.0366	0.0125
$E\left\{R^m\right\}$	0.0758	0.0758	0.0644	0.0508	0.0750
$E\left\{R^m - R^f\right\}$	0.0640	0.0641	0.0304	0.0142	0.0625
$\sigma\left\{R^f\right\}$	0.0229	0.0227	0.0084	0.0048	0.0256
$\sigma\left\{R^m\right\}$	0.1640	0.0156	0.0301	0.0163	0.0158
$\sigma\left\{R^m - R^f\right\}$	0.1618	0.0126	0.0222	0.0209	0.0108
$E\left\{\omega\right\}$	0.0291	0.0533	0.0399	0.0277	0.0524
$\sigma\left\{\omega\right\}$	0.0121	0.0027	0.0019	0.0009	0.0031
β		0.995	0.999	0.977	0.991
η		1.07	1.99	1.98	1.17
α		5.72	9.49	9.99	6.26

Note. The table reports the observed figures [column (1)] and the simulated values obtained from four different model specifications considering: the year-on-year convention and either consumer confidence [column (2)] or consumer sentiment [column (5)] as a demand shifter; the quarter-on-quarter convention and either consumer confidence [column (3)] or consumer sentiment [column (4)] as a demand shifter. The variables under scrutiny include the first two moments of the risk-free rate ($E\left\{R^f\right\}$ and $\sigma\left\{R^f\right\}$), market return ($E\left\{R^m\right\}$ and $\sigma\left\{R^m\right\}$), market premium ($E\{R^m - R^f\}$ and $\sigma\{R^m - R^f\}$), and dividend yield ($E\left\{\omega^m\right\}$ and $\sigma\left\{\omega^m\right\}$). The parameters β, η, and α respectively refer to the subjective discount factor, the intertemporal elasticity of substitution and the relative risk aversion coefficient.

Nevertheless, a comparison with Column (2) shows that the outcomes of these decompositions regarding the first group of statistics remain significantly worse than the baseline model's (and, hence, far away from the targets).

Before concluding, we check the sensitivity of our results to some changes in the procedure that we use to obtain the simulated predictions. We begin with considering an alternative measure for consumer confidence. The Conference Board's Consumer Confidence Index is often considered jointly with the University of Michigan's Consumer Sentiment Index.[21] Therefore,

[21] For a discussion on the historical reasons for this pairing, together with a detailed description of differences

Table 5.

Asset pricing statistics: confidence as probability shifter.

	(1)	(2)	(3)	(4)	(5)	(6)
		shifter: CC		shifter: CS		
Statistic	Data	with HF	w/o HF	with HF	w/o HF	Standard
$E\left\{R^f\right\}$	0.0118	0.0310	0.0385	0.0353	0.0384	0.0376
$E\left\{R^m\right\}$	0.0758	0.0528	0.0490	0.0500	0.0490	0.0498
$E\left\{R^m - R^f\right\}$	0.0640	0.0218	0.0105	0.0147	0.0106	0.0122
$\sigma\left\{R^f\right\}$	0.0229	0.0074	0.0091	0.0086	0.0091	0.0078
$\sigma\left\{R^m\right\}$	0.1640	0.0075	0.0155	0.0080	0.0132	0.0122
$\sigma\left\{R^m - R^f\right\}$	0.1618	0.0016	0.0215	0.0042	0.0208	0.0200
$E\left\{\omega\right\}$	0.0291	0.0305	0.0262	0.0279	0.0263	0.0269
$\sigma\left\{\omega\right\}$	0.0121	0.0022	0.0007	0.0016	0.0007	0.0008
β		0.968	0.975	0.968	0.974	0.971
η		2.00	0.97	2.00	0.98	1.22
α		10.0	9.99	9.99	9.99	9.99

Note. The table reports the observed figures [column (1)] and the simulated values obtained from five different models specifications considering: the year-on-year convention and consumer confidence [column (2)] or consumer sentiment [column (4)] as a probability shifter; or the quarter-on-quarter convention and consumer confidence [column (3)], consumer sentiment [column (5)], or no variable [column (6)] as a probability shifter. The variables under scrutiny include the first two moments of the risk-free rate ($E\left\{R^f\right\}$ and $\sigma\left\{R^f\right\}$), market return ($E\left\{R^m\right\}$ and $\sigma\left\{R^m\right\}$), market premium ($E\{R^m - R^f\}$ and $\sigma\{R^m - R^f\}$), and dividend yield ($E\left\{\omega^m\right\}$ and $\sigma\left\{\omega^m\right\}$). The parameters β, η, and α respectively refer to the subjective discount factor, the intertemporal elasticity of substitution and the relative risk aversion coefficient.

it seems natural to explore whether the results of our approach extend to using the Sentiment Index as a proxy for consumer confidence.

Table 4 summarizes the ensuing comparison. In order to aid the contrast with our previous findings visually, the first three columns are the same as in Table 3. Columns (4) and (5) instead report the predictions obtained using consumer sentiment as a demand shifter, respectively without and with habit formation. The table reveals that the two models deliver similar figures. A modest deterioration in replicating the first group of statistics accompanies a slight

and similarities between the two indices, see Bram and Ludvigson (1998).

Table 6.

Asset pricing statistics: tighter approach to VAR coefficients.

	(1)	(2)	(3)	(4)
Statistic	Data	Baseline CC + HF	CC only	CS only
$E\left\{R^f\right\}$	0.0118	0.0118	0.0326	0.0384
$E\left\{R^m\right\}$	0.0758	0.0758	0.0661	0.0490
$E\left\{R^m - R^f\right\}$	0.0640	0.0640	0.0335	0.0106
$\sigma\left\{R^f\right\}$	0.0229	0.0228	0.0074	0.0086
$\sigma\left\{R^m\right\}$	0.1640	0.0182	0.0296	0.0119
$\sigma\left\{R^m - R^f\right\}$	0.1618	0.0064	0.0222	0.0205
$E\left\{\omega\right\}$	0.0291	0.0530	0.0415	0.0263
$\sigma\left\{\omega\right\}$	0.0121	0.0030	0.0020	0.0007
β		0.991	0.999	0.974
η		1.08	2.00	0.98
α		3.79	9.21	9.99
$A_{x\theta}$		Yes	Yes	No
$A_{\theta x}$		No	Yes	No
$A_{\theta\theta}$		Yes	No	No

Note. The table reports the observed figures [column (1)] and the simulated values obtained from three different models specifications considering: consumer confidence as a demand shifter and either the year-on-year convention [column (2)] or the quarter-on-quarter convention [column (3)], or consumer sentiment as a demand shifter and the quarter-on-quarter convention [column (4)], with the VAR coefficients set to nil whenever the relevant estimated value are not significantly different from zero. The variables under scrutiny include the first two moments of the risk-free rate ($E\left\{R^f\right\}$ and $\sigma\left\{R^f\right\}$), market return ($E\left\{R^m\right\}$ and $\sigma\left\{R^m\right\}$), market premium ($E\{R^m - R^f\}$ and $\sigma\{R^m - R^f\}$), and dividend yield ($E\left\{\omega^m\right\}$ and $\sigma\left\{\omega^m\right\}$). The parameters β, η, and α respectively refer to the subjective discount factor, the intertemporal elasticity of substitution and the relative risk aversion coefficient.

improvement in matching the second one. We can then conclude that the same assessment of the baseline model also applies to the framework considering the Consumer Sentiment Index.

Several scholars deem consumer confidence as yielding information with exclusive regard to the availability of resources rather than consumers' proneness to consume. From this viewpoint, a more conservative approach considering the state variable merely as conditioning information for the events' probability distribution would appear more sensible. Table 5 reports the results of limiting consumer confidence role to measuring consumers' subjective beliefs concerning variations in available resources. Alongside the observed figures and simulated values from the standard model already reported in the previous tables [in Columns (1) and (6)], Table 5 illustrates the outcomes of using consumer confidence [respectively, sentiment] as a probability shifter, with or without considering habit formation, in Columns (2) and (3) [resp., (4) and (5)].

Contrasting Columns (2)-(5) to (6), it is immediate to note that the framework featuring the state variable as probability shifter yields only modest improvements relative to the standard model in replicating the observed statistics when habit formation is taken into account, and even a slight worsening when abstracting from habit persistence. These findings invariably translate in a significant deterioration in matching the first set of statistics, accompanied by a marginal amelioration in accounting for the second set. The entire set of results is obtained with poorer values of the calibrated preference parameters.

In order to allow for a more transparent comparison across the different model specifications, we produced the findings presented so far by letting the VAR coefficients take the relevant estimated values, regardless of whether these were significantly different from zero.[22] By affecting the transition probability matrix, this choice might alter the resulting simulated statistics. To rule out the possibility that the outcomes are seriously affected by our lenient approach to assigning values to VAR coefficients, Table 6 reports the results obtained by setting to nil those coefficients that are not statistically different from zero and shows that the simulated predictions remain virtually unchanged relative to those delivered by the baseline model and its specialization. In conjunction with the findings reported in Table 5, this result stresses the role of consumer confidence/sentiment as a demand shifter rather than a mere probability shifter in affecting asset prices' behavior.

5 Final remarks and conclusion

We have investigated the effects of including strong time preference linkages into a non-consumption-centric macro-finance model. We have done so by analyzing the effects of incorporating an exogenous state variable on the representative consumer's choice regarding consumption and investment decisions. The state variable has introduced two elements in the stochastic discount factors of the baliseline model: a demand shifter (consumer confidence) and year-on-year growth rates (on a quarterly data frequency). The year-on-year convention adopted to compute the growth rates may be interpreted as capturing potential habit formation; consumer confidence

[22] This robustness exercise is essential since the method proposed by Tauchen (1986) does not explicitly specify a clear-cut rule regarding the exclusion of estimated coefficients that are not statistically different from zero.

as the symmetric concept in an intertemporal perspective, in other words, as a way to capture potential utility from anticipation.

Our findings have indicated that the model compares favorably with the well-established approaches based on long-run risks and rare events in terms of calibrated preference parameters (governing the subjective discount factor, relative risk aversion, and intertemporal elasticity of substitution) as well as concerning a set of four statistics, namely the mean and standard deviation of the risk-free rate, the mean of the market return and the market risk premium. In contrast, the model underperforms concerning a second set of four statistics, namely the standard deviation of the market return and risk premium, and the mean and volatility of the dividend yield.

We have considered three other model specifications to evaluate the impact of the two elements in isolation and the performance of the model that abstracts from both of them. We have found that disregarding either or both elements results in an acute deterioration of the model's performance. In relative terms, our results suggest that dropping consumer confidence is slightly less detrimental than excluding habit persistence or discarding both elements. Furthermore, we have examined the effect of replacing the Consumer Confidence Index with the Consumer Sentiment Index to measure consumer confidence. Our results suggest that the models' performance using the two alternative measures is fairly comparable. Finally, two additional robustness checks point out that one should not regard confidence indicators merely as probability shifters; instead, evidence suggests that they play the role of demand shifters.

By matching the observed risk premium and the risk-free rate volatility, the baseline model stands as a serious candidate to offer an alternative rationale for asset prices' behavior. Nevertheless, the inaptitude to produce sufficient variability across the different conditioning states suggests that the model cannot fully account for the pricing kernel's dynamics captured by the approaches based on long-run risks and rare events.

A Appendix

A.1 Conventional wisdom: anecdotal support

Examining Google search engine queries supports the relevant role collectively given by the financial markets, the media, and the business community to consumer confidence indicators. Coupling the terms "finance" and "consumer confidence" as a query returns 5,870,000 hits. This figure markedly outweighs queries coupling "finance" and some other references to notable concepts related to financial economics, such as "habit formation" or "economic disaster," which return 573,000 and 1,200,000 hits, respectively. It also compares favorably with queries coupling "finance" and broader concepts, such as "economic uncertainty," which returns 2,850,000 hits. To put these figures in the proper perspective, note that a query coupling "finance" and "economic crisis" returns 29,900,000 hits; "finance" and "financial markets" 41,900,000. (Data retrieved by the authors on October 11, 2020.)

Illustrative examples of the popular conceptualization according to which higher confidence would induce consumers to be more prone to consume include, among countless others, statements like: "When consumer confidence is high, consumers make more purchases. When confidence is low, consumers tend to save more and spend less." (en.wikipedia.org/wiki/Consumer_confidence); "A high level of consumer confidence will encourage a higher marginal propensity to consume." (economicshelp.org/blog/6544/economics/uk-consumer-confidence-2); "In the most simplistic terms, when their confidence is trending up, consumers spend money, indicating the sustainability of a healthy economy." (investopedia.com/insights/understanding-consumer-confidence-index).

Regarding the use of the year-on-year convention in computing growth rates, we note that coupling the terms "finance" and "year on year" as a query returns 23,400,000 hits, a much more prominent figure than those produced by coupling the terms "finance" and "quarter on quarter" (756,000 hits) or "month on month" (1,790,000 hits). It could be argued that these figures reflect the relative use of the data frequency to which they respectively refer. However, pairing the term "finance" with "quarterly data" and with "monthly data" return a number of hits (3,490,000 and 15,100,000, respectively) by comparison far higher than the relevant previous queries, whereas pairing "finance" with "annual data" a drastically lower figure (4,204,000 hits, which turn to 4,590,000 if one also considers those obtained by coupling "finance" and "yearly data").

Note also that coupling the terms "finance" and "resolution of uncertainty" as a query in the Google search engine returns 3,390,000 hits, thereby faring almost as well as pairing the term "finance" with "consumer confidence."

A.2 Derivation of the stochastic discount factor

Except for the preference specification (1), our framework is analogous to the Epstein-Zin-Weil (EZW) model: consumers' preferences are represented by a recursive utility function; two assets, one risk-free and the other state-contingent, are traded; free portfolio formation and the law of one price hold.

The representative consumer maximizes lifetime utility subject to the budget constraint

$$(p_{t+1} + y_{t+1})\, z_t + b_t \geq c_t + p_{t+1} z_{t+1} + q_{t+1} b_{t+1}$$

where b is the bond holding, q is the bond price, z is the stock holding, y is the stock dividend and p is the stock price. To ease notation, we denote the aggregate state with $s = (\kappa, \gamma, c, x)$. The variables involved in the determination of the state are levels and growth factors of the state variable and consumption, respectively related by the two equalities

$$\kappa_{t+1} = \gamma_{t+1}\kappa_t \text{ and } c_{t+1} = x_{t+1}c_t$$

with the pair (γ, x) following a Markov chain. Keeping this in mind, the representative consumer's dynamic program can be formalized as

$$v\{z_t, b_t, s_t\} = \max_{c_t, z_{t+1}, b_{t+1}} \left[(1-\beta)(\kappa_t c_t)^\rho + \beta\mu_{s_t}\{v\{z_{t+1}, b_{t+1}, s_{t+1}\}\}^\rho\right]^{\frac{1}{\rho}}$$

subject to

$$(p\{s_t\} + y_t) z_t + b_t \geq c_t + p\{s_t\} z_{t+1} + q\{s_t\} b_{t+1}$$

where $\mu_s\{\cdot\}$ is the certainty equivalent conditional on the state s; likewise, the stock and bond prices, $p\{s\}$ and $q\{s\}$, are also conditional on the state s; $v\{\cdot\}$ is the representative consumer's value function conditional on the asset holdings z and b as well as on the state s.

Denote $W\{c, \mu\} = [(1-\beta)(\kappa c)^\rho + \beta\mu^\rho]^{\frac{1}{\rho}}$, and note that the partial derivatives are

$$\begin{aligned} W_c\{c,\mu\} &= \frac{1}{\rho}[(1-\beta)(\kappa c)^\rho + \beta\mu^\rho]^{\frac{1}{\rho}-1}(1-\beta)\rho\kappa^\rho c^{\rho-1} = (1-\beta)(W\{c,\mu\})^{1-\rho}\kappa^\rho c^{\rho-1} \\ W_\mu\{c,\mu\} &= \frac{1}{\rho}[(1-\beta)(\kappa c)^\rho + \beta\mu^\rho]^{\frac{1}{\rho}-1}\beta\rho\mu^{\rho-1} = \beta(W\{c,\mu\})^{1-\rho}\mu^{\rho-1} \end{aligned}$$

The partial derivative of μ_{s_t} with respect to z_{t+1} is

$$\begin{aligned} &\frac{\partial}{\partial z_{t+1}}\mu_{s_t}\{v\{z_{t+1}, b_{t+1}, s_{t+1}\}\} = \\ &\qquad \left(\mu_{s_t}\{v\{z_{t+1}, b_{t+1}, s_{t+1}\}\}\right)^\alpha E_t\left\{[v\{z_{t+1}, b_{t+1}, s_{t+1}\}]^{-\alpha}\frac{\partial}{\partial z_{t+1}}v\{z_{t+1}, b_{t+1}, s_{t+1}\}\right| s_t\right\} \end{aligned}$$

The first-order condition (FOC) for the choice of z_{t+1} is

$$\begin{aligned} &W_c\left\{c_t, \mu_{s_t}\{v\{z_{t+1}, b_{t+1}, s_{t+1}\}\}\right\} p\{s_t\} = \\ &\qquad W_\mu\left\{c_t, \mu_{s_t}\{v\{z_{t+1}, b_{t+1}, s_{t+1}\}\}\right\}\frac{\partial}{\partial z_{t+1}}\mu_{s_t}\{v\{z_{t+1}, b_{t+1}, s_{t+1}\}\} \end{aligned}$$

which we can write as

$$\begin{aligned} &(1-\beta)(\kappa_t)^\rho(c_t)^{\rho-1} p\{s_t\} = \\ &\qquad \beta\mu_{s_t}\{v\{z_{t+1}, b_{t+1}, s_{t+1}\}\}^{\rho-1+\alpha} E_t\left\{(v\{z_{t+1}, b_{t+1}, s_{t+1}\})^{-\alpha}\frac{\partial}{\partial z_{t+1}}v\{z_{t+1}, b_{t+1}, s_{t+1}\}\right| s_t\right\} \end{aligned}$$

We can use an envelope argument to get an expression for the derivative of v with respect to z. From the budget constraint we have

$$\frac{\partial}{\partial z} c\{z,\cdot\} = p\{s\} + y$$

At state (z_t, b_t, s_t), the derivative is given by

$$\begin{aligned}\frac{\partial}{\partial z_t} v\{z_t, b_t, s_t\} &= (1-\beta)\left(W\left\{c_t, \mu_{s_t}\{v\{z_{t+1}, b_{t+1}, s_{t+1}\}\}\right\}\right)^{1-\rho}(\kappa_t)^{\rho}(c_t)^{\rho-1}(p\{s_t\}+y_t) \\ &= (1-\beta)(v\{z_t, b_t, s_t\})^{1-\rho}(\kappa_t)^{\rho}(c_t)^{\rho-1}(p\{s_t\}+y_t)\end{aligned}$$

We now advance this expression one period and plug it into the right-hand side of the FOC to get the first-order condition for the holdings of the stock

$$\begin{aligned}(\kappa_t)^{\rho}(c_t)^{\rho-1} p\{s_t\} = \beta\mu_{s_t}\{v\{z_{t+1}, b_{t+1}, s_{t+1}\}\}^{\rho-1+\alpha}\cdot \\ E_t\left\{(v\{z_{t+1}, b_{t+1}, s_{t+1}\})^{1-\rho-\alpha}(\kappa_{t+1})^{\rho}(c_{t+1})^{\rho-1}(p\{s_{t+1}\}+y_{t+1})\Big|\, s_t\right\}\end{aligned} \tag{15}$$

The first-order condition concerning the riskless asset is analogous and obtained simply by plugging in $q\{s_t\}$ for $p\{s_t\}$ and 1 for the payoff $p\{s_{t+1}\} + y_{t+1}$, obtaining

$$\begin{aligned}&(\kappa_t)^{\rho}(c_t)^{\rho-1} q\{s_t\} = \\ &\qquad \beta\mu_{s_t}\{v\{z_{t+1}, b_{t+1}, s_{t+1}\}\}^{\rho-1+\alpha} E_t\left\{(v\{z_{t+1}, b_{t+1}, s_{t+1}\})^{1-\rho-\alpha}(\kappa_{t+1})^{\rho}(c_{t+1})^{\rho-1}\Big|\, s_t\right\}\end{aligned}$$

Imposing equilibrium (consumption must equal dividends, i.e., $c = y$; the representative household constantly holds all the stock, i.e. $z = 1$; but no bond, i.e., $b = 0$) and rearranging the model's asset pricing formulas for the equity price becomes

$$p\{s_t\} = E_t\left\{\beta\left(\frac{V\{s_{t+1}\}}{\mu_{s_t}\{V\{s_{t+1}\}\}}\right)^{1-\rho-\alpha}(\gamma_{t+1})^{\rho}(x_{t+1})^{\rho-1}(p\{s_{t+1}\}+y_{t+1})\Bigg|\, s_t\right\} \tag{16}$$

where to simplify notation we let $V\{s\} = v\{1, 0, s\}$, representing the representative consumer's value function in equilibrium. The right-hand side of equation (3) corresponds to the first four terms of the expectation operator's argument.

Iterative procedure

Since the pair (γ, x) is assumed to follow a Markov chain and lifetime utility is homogeneous of degree one in κc, the SDF depends only on (γ_t, x_t) and (γ_{t+1}, x_{t+1}), with γ_t and x_t appearing just in the conditioning of the certainty equivalent. For some function ϕ, the equilibrium value function can be therefore written as

$$V\{s_t\} = \phi\{\gamma_t, x_t\}\kappa_t y_t \tag{17}$$

Plugging this expression into (2), we can rewrite the certainty equivalent operator as

$$\mu_t\{V\{s_{t+1}\}\} = \left(E_t\left\{\left(\phi\{\gamma_{t+1}, x_{t+1}\}\gamma_{t+1}x_{t+1}\right)^{1-\alpha}\right\}\right)^{\frac{1}{1-\alpha}}\kappa_t y_t$$

The third term in (3) then becomes

$$\left(\frac{V\{s_{t+1}\}}{\mu_{s_t}\{V\{s_{t+1}\}\}}\right)^{1-\rho-\alpha} = \left(\frac{\phi\{\gamma_{t+1}, x_{t+1}\}\gamma_{t+1}x_{t+1}}{\mu_{\gamma_t,x_t}\{\phi\{\gamma_{t+1}, x_{t+1}\}\gamma_{t+1}x_{t+1}\}}\right)^{1-\rho-\alpha}$$

where $\mu_{\gamma_t,x_t}\{\phi\{\gamma_{t+1}, x_{t+1}\}\gamma_{t+1}x_{t+1}\} \equiv \left(E_t\left\{\left(\phi\{\gamma_{t+1}, x_{t+1}\}\gamma_{t+1}x_{t+1}\right)^{1-\alpha}\right\}\right)^{\frac{1}{1-\alpha}}$, which in turn implies that the SDF reads

$$m\{\gamma_t, \gamma_{t+1}, x_t, x_{t+1}\} = \beta(x_{t+1})^{\rho-1}(\gamma_{t+1})^{\rho}\left(\frac{\phi\{\gamma_{t+1}, x_{t+1}\}\gamma_{t+1}x_{t+1}}{\mu_{\gamma_t,x_t}\{\phi\{\gamma_{t+1}, x_{t+1}\}\gamma_{t+1}x_{t+1}\}}\right)^{1-\rho-\alpha}$$

In this expression, x_{t+1} and ϕ are vectors, while m is the matrix

$$m\{j,k\} = \beta(x\{k\})^{\rho-1}(\gamma\{k\})^{\rho}\left(\frac{\phi\{k\}\gamma\{k\}x\{k\}}{\mu\{j\}}\right)^{1-\rho-\alpha} \tag{18}$$

where j denotes the current state, k the future state and

$$\mu\{j\} = \left(\sum_{k=1}^{S}\pi\{j,k\}\left(\phi\{k\}\gamma\{k\}x\{k\}\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha}} \tag{19}$$

with $\pi\{j,k\}$ indicating the transition probability from state j to state k.

Together with (16), (18) and (19) entail that the equity price is homogeneous in κc. Using (16), the price/dividend ratio, defined as $\omega\{s\} = p\{s\}/y$, can be written as

$$\omega\{j\} \equiv \frac{p\{j\}}{y\{j\}} = \sum_{k=1}^{S}\pi\{j,k\}m\{j,k\}x\{k\}(1+\omega\{k\})$$

The bond price is simply

$$q\{j\} = \sum_{k=1}^{S}\pi\{j,k\}m\{j,k\}$$

We need to compute the matrix $m = m\{j,k\}$ to solve these equations. This task requires the calculation of the function ϕ. Here is where it becomes necessary to resort to the iterative procedure. Recall that $\phi\{\gamma, x\}\kappa y$ is the value function in equilibrium, which in turn represents the representative consumer's maximized lifetime utility. We can therefore write

$$\phi\{\gamma_t, x_t\}\kappa_t y_t = \left[(1-\beta)(\kappa_t y_t)^\rho + \beta\left(\mu_{\gamma_t, x_t}\left\{\phi\left\{\gamma_{t+1}, x_{t+1}\right\}\kappa_{t+1} y_{t+1}\right\}\right)^\rho\right]^{\frac{1}{\rho}}$$

which, dividing both sides by $\kappa_t y_t$, using $\kappa_{t+1} = \gamma_{t+1}\kappa_t$, $y_{t+1} = x_{t+1} y_t$ and the homogeneity of μ, becomes

$$\phi\{\gamma_t, x_t\} = \left[1 - \beta + \beta\left(\mu_{\gamma_t, x_t}\left\{\phi\left\{\gamma_{t+1}, x_{t+1}\right\}\gamma_{t+1} x_{t+1}\right\}\right)^\rho\right]^{\frac{1}{\rho}}$$

For the generic state i, this expression corresponds to the vector

$$\phi\{i\} = \left[1 - \beta + \beta(\mu\{i\})^\rho\right]^{\frac{1}{\rho}} \tag{20}$$

with $\mu\{\cdot\}$ as in (19). To solve for ϕ, we treat the last two equations as a mapping that, at the k-th iteration, takes the vector ϕ^{k-1} into a new vector ϕ^k. Specifically: given ϕ^{k-1}, we first use (19) to generate a vector of certainty equivalents μ^k; we then use μ^k to obtain ϕ^k using (20). This two-step procedure is repeated iteratively until the change in ϕ produced by successive iterations is sufficiently small to be considered negligible.

A.3 Calibration of Markov chain and variables' realizations

In order to examine the quantitative aspects of asset pricing, we use a finite-state discrete Markov chain for the state variables. Specifically, we apply the method developed by Tauchen (1986) for choosing values for the state variables and the transition probabilities so that the resulting finite-state Markov chain mimics an underlying continuous-valued autoregression closely. The motivation for the method is the well-known fact that captures the statistical properties of the time series involved in the analysis adequately (after an adjustment for trend).

We begin by characterizing the vector autoregressive model. Let the growth rates of the M variables involved in the analysis be[23]

$$\begin{aligned} &g_t^v \equiv v_{t+1}/v_t, \text{ for } v = v_1, v_2, ..., v_M \text{ and } t = 1, 2, ..., T \\ &g_t \equiv [g_t^{v_1}, g_t^{v_2}, ..., , g_t^{v_M}] \text{ for } t = 1, 2, ..., T \\ &g \equiv [g_1, g_2, ..., g_T]' \end{aligned} \tag{21}$$

where g_t is a $1 \times M$ vector collecting the growth rates at time t, and g a $T \times M$ matrix.

[23]The number of variables equals the generic value M because our analysis requires the model to be solved for the consumption growth in isolation (one variable) as well as for the consumption growth rate in conjunction with the state variable growth rate (two variables).

Furthermore, let

$$\begin{array}{l} E\left(g^{v}\right) \equiv \frac{1}{T}\sum_{t=1}^{T} g_t^v, \text{ for } v = v_1, v_2, ..., v_M \\ E\left(g\right) \equiv \left[E\left(g^{v_1}\right), E\left(g^{v_2}\right), ..., E\left(g^{v_M}\right)\right] \end{array} \tag{22}$$

where $E\left(g\right)$ is a $1 \times M$ vector collecting the unconditional average growth rates. Finally, let

$$y_t \equiv g_t - E\left(g\right) \tag{23}$$

be generated by the vector autoregressive (VAR) process

$$y_t = Ay_{t-1} + \epsilon_t \tag{24}$$

with the VAR error term covariance matrix represented by

$$var\left(\epsilon_t\right) = \Sigma_\epsilon \tag{25}$$

a diagonal $M \times M$ matrix, where A is the $M \times M$ matrix of VAR coefficients and ϵ_t is the $1 \times M$ vector of VAR white noise error terms at time t, with the v-th element denoted by ϵ_t^v. It is assumed that the elements of ϵ_t are mutually independent, each with distribution $\Pr\left[\epsilon_t^v \leq u\right] = Z\left(u/\sigma\left(\epsilon^v\right)\right)$, where Z is the cumulative distribution of a standardized Gaussian process and $\sigma\left(\epsilon^v\right)$ is the standard deviation of the VAR error term ϵ^v.

We now turn to develop the structure of the finite-state discrete model. Let $\tilde{y}_t$ denote the approximating Markov chain vector for y_t in (24). Each component $\{\tilde{y}_t^v\}_{v=v_1,...,v_M}$ takes on one of N values

$$\begin{array}{l} \bar{y}_1^v < \bar{y}_2^v < ... < \bar{y}_N^v, \text{ for } v = v_1, v_2, ..., v_M \\ \bar{y}^v \equiv \left[\bar{y}_1^v, \bar{y}_2^v, ..., \bar{y}_N^v\right], \text{ for } v = v_1, v_2, ..., v_M \\ \bar{y} \equiv \left[\bar{y}^{v_1}, \bar{y}^{v_2}, ..., \bar{y}^{v_M}\right]' \end{array} \tag{26}$$

where the generic value $\bar{y}_l^v$ is indexed by $l = 1, 2, ..., N$ and $\bar{y}$ is a $M \times N$ matrix.

A method for selecting the values of the components of $\bar{y}^v$ for each $v = v_1, v_2, ..., v_M$ is to let $\bar{y}_1^v$ and $\bar{y}_N^v$ be respectively minus and plus a small integer m times the unconditional standard deviation of y_t^v, with the remaining components satisfying

$$\bar{y}_{l+1}^v = \bar{y}_l^v + w^v, \text{ for } v = v_1, v_2, ..., v_M \text{ and } l = 1, 2, ..., N-1 \tag{27}$$

where

$$w^v = 2m\,\sigma\left(y^v\right) / \left(N-1\right), \text{ for } v = v_1, v_2, ..., v_M \tag{28}$$

The $\{\sigma\left(y^v\right)\}_{v=x,\gamma}$ are the square roots of the diagonal elements of the matrix Σ_y that satisfies

$\Sigma_y = A\Sigma_y A' + \Sigma_\epsilon$, which can be found by iterating

$$\Sigma_y(r) = A\Sigma_y(r-1)A' + \Sigma_\epsilon \tag{29}$$

with convergence as $r \to \infty$ guaranteed so long as (24) is stationary.

There are N^M possible states for the system. Enumerate these states using the index $i = 1, 2, ..., N^M$. Let

$$\begin{aligned} &\bar{l}(i) \equiv \left[\bar{l}^{v_1}(i), \bar{l}^{v_2}(i), ..., \bar{l}^{v_M}(i)\right]', \text{ for } i = 1, 2, ..., N^M \\ &L \equiv \left[\bar{l}(1), \bar{l}(2), ..., \bar{l}\left(N^M\right)\right] \end{aligned} \tag{30}$$

where L is a $M \times N$ matrix, and $\bar{l}(i)$ is a $M \times 1$ vector of integers associated with state i such that, when the system is in state i at any given time t, the components of $\tilde{y}_t \equiv \tilde{y}(i)$ assume the values

$$\tilde{y}^v(i) = \bar{y}^v_{q^v} \text{ for } v = v_1, v_2, ..., v_M$$

where $q^v = \bar{l}^v(i)$. As a result, when the system is in state i at any given time t, $\tilde{y}_t = \bar{y}^i = \left[\bar{y}^{v}_{q^{v_1}}, \bar{y}^{v2}_{q^{v_2}}, ..., \bar{y}^{v_M}_{q^{v_M}}\right]'$. We sort the states in such a way that, as the state index increases, the value of the component $\tilde{y}^{v_1}$ varies only after it has been matched with each value of the component $\tilde{y}^{v_2}$, which in turn varies only after it has been matched with each value of the component $\tilde{y}^{v_2}$, and so forth.

We wish to calculate the transition matrix $\pi(j,k) = \Pr\left[\tilde{y}_t = \bar{y}^k \,|\, \tilde{y}_{t-1} = \bar{y}^j\right]$. Let

$$\mu \equiv \left[\mu^{v_1}, \mu^{v_2}, ..., \mu^{v_M}\right]' = A\tilde{y}(j) \tag{31}$$

denote the impact of the lagged variables on of the realization of $\tilde{y}_t$ conditional on the state at time $t-1$ being j. For each v, let $h^v(j,l) = \Pr\left[\tilde{y}^v_t = \bar{y}^v_l \,|\, \text{state } j \text{ at } t-1\right]$, for $v = v_1, v_2, ..., v_M$, be the marginal probability that the v-th component of $\tilde{y}_t$ takes the value $\bar{y}^v_l$ conditional on observing the state j at time $t-1$; specifically, we define

$$\begin{aligned} h^v(j,l) \ &= Z\left(\bar{y}^v_l - \mu^v + w^v/2\right) - Z\left(\bar{y}^v_l - \mu^v - w^v/2\right) && \text{if } 2 \le l \le N-1 \\ &= Z\left(\bar{y}^v_1 - \mu^v + w^v/2\right) && \text{if } l = 1 \\ &= 1 - Z\left(\bar{y}^v_N - \mu^v - w^v/2\right) && \text{if } l = N \end{aligned} \tag{32}$$

Given (32), the transition probabilities $\pi(j,k) = \Pr[\text{in state } k \,|\, \text{ in state } j]$ are, by independence of the ϵ^v, the products of the appropriate h^v,

$$\pi(j,k) = \prod_{v=x,\gamma} h^v\left(j, \bar{l}^v(k)\right), \text{ for } j,k = 1,2,...,N^M,$$
$$\pi = \begin{bmatrix} \pi(1,1) & \pi(1,2) & ... & \pi\left(1,N^M\right) \\ \pi(2,1) & \pi(2,2) & ... & \pi\left(2,N^M\right) \\ ... & ... & ... & ... \\ \pi\left(N^M,1\right) & \pi\left(N^M,2\right) & ... & \pi\left(N^M,N^M\right) \end{bmatrix} \tag{33}$$

where π is a $N^M \times N^M$ matrix.

The $1 \times N^M$ vector of unconditional probabilities is identified by any (e.g., the first) row of the matrix obtained by raising π in (33) to a power χ large at will,

$$p = I_{N^M} \cdot \pi^\chi \tag{34}$$

where $I_{N^M} \equiv [1,0,...]$ is a $1 \times N^M$ vector.

The realizations of the Markov chain corresponding to the future states characterizing the columns of π and p are identified using the matrix $\bar{y}$ in (26) in conjunction with the matrix L in (30). Specifically, for state i and variable v, the value $l^v(i)$ in L identifies the column of $\bar{y}$ storing, at the v-th row, the Markov chain realization relative to v in i. The value so identified must be added to the m-th column of the vector in (22) to obtain the i-th state realization of variable v's growth rate,

$$\begin{aligned} &\bar{g}^v(i) = \bar{y}^v\left(l^v(i)\right) + E\left(g^v\right), \text{ for } v = v_1, v_2, ..., v_M \text{ and } i = 1,2,...,N^M \\ &\bar{g}^v = \left[\bar{g}^v(1), \bar{g}^v(2), ..., \bar{g}^v\left(N^M\right)\right]', \text{ for } v = v_1, v_2, ..., v_M \\ &\bar{g} = \left[\bar{g}^{v_1}, \bar{g}^{v_2}, ..., \bar{g}^{v_{N^M}}\right] \end{aligned} \tag{35}$$

where $\bar{g}$ is a $N^M \times M$ matrix.

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