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By force of confidence:
A media-based approach to asset pricing[*]

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Abstract

Casual observation suggests that financial practitioners resort to different model specifications than those populating the macro-finance literature. We investigate whether there is an advantage in doing so by comparing the outcomes of the well-established long-run risk model of Bansal and Yaron (2004) with those obtained by a consumption-based model incorporating two popular notions in the financial sector. Namely, (i) the conventional wisdom "confidence makes households feel richer, hence willing to consume more" and (ii) the year-on-year convention to compute growth rates. The model features a recursive non-separable utility defined over two stochastic variables, consumption and consumer confidence. We find that the model compares favorably with Baron and Yansal (2004) in explaining the mean of the market return and the first two moments of the risk-free rate, whereas it falls short in rationalizing the first two moments of the price-dividend ratio and the volatility of the market return.

JEL Classification: G12, E21

Keywords: Asset Pricing, Consumer Confidence, Habit Persistence, Utility from Anticipation, Year-on-Year Growth

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1 Introduction

In the last four decades, macro-finance models have gone a long way to explain the concurrent behavior of consumption growth, risk-free rate, equity return, and price-dividend ratio. Several breakthroughs have been achieved by considering preferences for early resolution of uncertainty, habit formation, uncertainty about the state of the economy, and macroeconomic events resulting in rare disasters, to cite some of the most influential explorations.[1] Little is known about the extent to which practitioners use these new findings when planning their financial strategies. Indeed, casual observation suggests that the typical focus is on popular information sources and tools that part of the academic world would find awkward or disregard. Motivated by such considerations, we investigate whether there is an advantage in using some of these information sources and tools in an asset pricing framework. We do so by formalizing a simple but internally consistent and methodologically rigorous approach to macro-finance, in which we incorporate a few popular elements in the financial sector that yield interesting time preference linkages. We then compare our findings with well-established results in the literature.

We develop a consumption-based asset pricing model taking into account two prominent aspects that consistently emerge from several contributions in empirical finance as well as macroeconomic, business, and political news. The first aspect concerns information sources. In the financial literature, a string of empirical contributions considers confidence indicators' potential role as conditioning information in factor asset pricing models.[2] The financial markets, the media and the business community hold consumer confidence indicators in high regard when assessing or forecasting economic and financial conditions.[3] Confidence is generally interpreted as an indicator of prospective changes in consumers' income or wealth. Higher confidence, the typical story goes, signals better economic conditions; this induces consumers to feel richer and, accordingly, more prone to consume.[4] We let this conventional wisdom guide our modeling

[1] Comprehensive surveys of the macro-finance literature can be found in Mehra (2012), Ludvigson (2013), Campbell (2015), Constantinides (2017), and Cochrane (2017).

[2] Examples include Ho and Hung (2009) and Bathia and Bredin (2018), who include investor sentiment as conditioning information in factor asset pricing models to study the relevance of the size, value, liquidity and momentum effects on individual stocks returns; Lemmon and Portniaguina (2006), who investigate the time-series relationship between consumer confidence and the returns of small stocks; Chung, Hung and Yeh (2012), who study the potential asymmetry of the predictive power of investor sentiment on stock returns during economic expansions and recessions.

[3] Influential papers corroborate this view (see, e.g., Barsky and Sims, 2012). Examining Google queries also supports this collective perspective. Coupling the terms "finance" and "consumer confidence" as a query returns 5,870,000 hits. This figure markedly outweighs queries coupling "finance" and some other references to notable concepts related to financial economics, such as "habit formation" or "economic disaster," which return 573,000 and 1,200,000 hits, respectively. It also compares favorably with queries coupling "finance" and broader concepts, such as "economic uncertainty," which returns 2,850,000 hits. To put these figures in the right perspective, note that a query coupling "finance" and "economic crisis" returns 29,900,000 hits; "finance" and "financial markets" 41,900,000. (Data retrieved by the authors on October 11, 2020.)

[4] Illustrative examples of this popular conceptualization include, among countless others, statements like: "When consumer confidence is high, consumers make more purchases. When confidence is

strategy. Consumer confidence plays the role of an exogenous state variable, thereby signaling a regime of favorable or critical attitude towards consumption by influencing its marginal utility.

The second aspect is practical: growth rates are computed using the year-on-year convention, regardless of the (often higher) frequency characterizing the relevant data.[5] While fairly common in other branches of the macroeconomic literature, this method is hardly ever used in asset pricing. A rare exception is Jagannathan and Wang (2007), who argue that the "use of calendar year returns avoids the need to explain various well-documented seasonal patterns in stock returns, [...] [and] also attenuates the errors that may arise due to ignoring the effect of habit formation on preferences" (p. 1626). Importantly, this statement points out that our approach implicitly takes habit persistence into account since the year-on-year convention causes the Euler equation to comprise several higher-frequency growth factor lags, albeit in the compounded form of a yearly growth factor. The rest of the model draws on the basic framework (hereafter, EZW) proposed by Epstein and Zin (1989) and Weil (1989). As such, our model features consumers' preference over the timing of resolution of uncertainty, too.[6] Owing to this feature, the model shares some characteristics with the one (hereafter, BY) developed by Bansal and Yaron (2004), which we elect as the literature's benchmark to comparatively assess our model's outcomes.

The model's core mechanism is analogous to the one exploited by the standard Lucas (1978) "tree" model. If the asset payoffs covary positively with the consumption growth process, then the resulting negative relationship between asset returns and the stochastic discount factor (SDF) drives the expected market premium upwards. The newly introduced elements on the marginal utility of consumption adds another layer of variation to the core mechanism. In the presence of a positive correlation between consumer confidence innovation and consumption growth, the impact of confidence on marginal utility reinforces the SDF's deviation from its mean across states of nature. The time linkages generated by habit persistence further strengthens this effect. Therefore, the SDF volatility is larger when the model takes consumer confidence and year-on-year growth rates into account. This outcome suggests that the novel source of variability acts as a magnifier of asset prices' response to consumption growth fluctuations.

low, consumers tend to save more and spend less." (`en.wikipedia.org/wiki/Consumer_confidence`); "A high level of consumer confidence will encourage a higher marginal propensity to consume." (`economicshelp.org/blog/6544/economics/uk-consumer-confidence-2`); "In the most simplistic terms, when their confidence is trending up, consumers spend money, indicating the sustainability of a healthy economy." (`investopedia.com/insights/understanding-consumer-confidence-index`).

[5] Resorting once again on Google search engine's results, we note that coupling the terms "finance" and "year on year" as a query returns 23,400,000 hits, a much larger figure than those produced by coupling the terms "finance" and "quarter on quarter" (756,000 hits) or "month on month" (1,790,000 hits). It could be argued that these figures reflect the relative use of the data frequency to which they respectively refer. However, the pair of terms ("finance", "quarterly data") and ("finance", "monthly data") return a number of hits (3,490,000 and 15,100,000, respectively) by comparison far larger than the relevant previous queries, whereas the pair ("finance", "yearly data") a drastically lower figure (386,000 hits, which turn to 4,590,000 if one considers those obtained by coupling "finance" and "annual data").

[6] Coupling the terms "finance" and "resolution of uncertainty" as a query in the Google search engine fares almost as well as the pair of terms ("finance", "consumer confidence"), returning 3,390,000 hits.

As a result, our approach is suited to replicate the observed financial statistics with a lower consumption growth volatility than the one required by the EZW model.

Our modeling strategy is parsimonious, allowing us to retain the same three-parameter preference specification as in EZW and BY. We calibrate all these parameters by matching the values of three out of six simulated statistics (namely, the first two moments of the risk-free rate, market return and price-dividend ratio) with the relevant figures observed in the data. We then have three untargeted moments, which we use to assess the performance of the model quantitatively. On the one hand, our findings indicate that our framework compares favorably with the BY model with regards to explaining a subset of statistics, as it accurately accounts for the observed mean excess return and riskless rate, as well as the volatility of the latter, with reasonable values of the three preference parameters. Notably, this is why we can calibrate the full set of preference parameters against observed statistics, an undertaking hardly ever engaged in by the existing contributions in the literature. On the other hand, the model falls short in rationalizing the second (complementary) subset of statistics, i.e., the first two moments of the price-dividend ratio and the volatility of the market return.

Our analysis further shows that one should consider consumer confidence and the year-on-year convention jointly. The outcomes of the model significantly worsen once we either drop from its specification the consumer confidence as a state variable, or we refrain from using the year-on-year convention to compute growth rates, or both (which corresponds to a version of the EZW model, here a special case of our approach). This finding is suggestive of a *persistent* role for consumer confidence in influencing the SDF, with lagged signals concurring with the current one in shaping the asset prices' behavior.

The paper is organized as follows. The remaining of this section reviews the contributions in the literature that are more closely related to our investigation. Section 2 illustrates the consumption-based asset pricing model with preferences augmented with an exogenous state variable; it also shows under which specifications of the state variable the stochastic discount factor comes to depend on year-on-year growth rates, in an environment characterized by higher-frequency time intervals. Section 3 describes the data that we use for our quantitative exercises, details the procedures we adopt to calibrate the model and discusses the resulting findings. Section 4 concludes. The appendix contains the most relevant mathematical derivations.

Related literature

The paper relates to several studies that investigate the relationship between consumer confidence and consumption growth. Ludvigson (2004) reports that these studies are motivated by empirical evidence suggesting that consumer confidence predicts consumption growth, over and above other commonly used economic indicators. Acemoglu and Scott (1994) rationalize the observed correlation by positing that consumer confidence variations reflect alterations in the degree of economic uncertainty. As such, these variations might alter precautionary savings motives, owing to changes in the forecast variance of consumption. The authors provide evidence that consumer confidence not only covaries with forecast variance, which suggests a positive link between saving and uncertainty.[7] It also correlates with consumption growth. Building on the latter observation, we show that consumer confidence variations may affect the SDF in the absence of time-varying consumption growth volatility.[8]

Carroll, Fuhrer and Wilcox (1994) argue that the observed correlation between consumer confidence and consumption growth suggests a potential role for habit formation.[9] As such, our paper also relates to papers that incorporate habit persistence through non-time-separable preferences. Habit can be external (Abel, 1990; Campbell and Cochrane, 1999), merely acting as a reference point, or internal, letting consumers' current marginal utility depend on their own past consumption choice (Constantinides, 1990). Our framework implicitly incorporates external habit formation. As already mentioned, our findings indicate that both consumer confidence and habit formation are individually instrumental in obtaining a reasonable account of macro-finance facts. We thus contribute to this literature by providing evidence that the two variables play distinctive roles in explaining asset prices.

More broadly, our paper relates to contributions that enrich the instantaneous utility function with additional arguments governing consumers' time preference. These encompass models that include habit formation as well as models that incorporate utility from anticipation.[10] Campbell and Cochrane (1999, p. 208) eloquently state that habit formation "captures a fundamental feature of psychology: repetition of a stimulus diminishes the perception of the stimulus and responses to it." Utility of anticipation represents the symmetric stance in an intertemporal perspective: the anticipation of a future stimulus alters the perception of current stimuli and responses to them. From this perspective, one may interpret habit formation as a measure of the impact on the current marginal utility of consumption of past events' reminiscence; consumer confidence of the anticipation of *future* conditions. In a seminal paper, Loewenstein (1987) explicitly links anticipation to *internal* factors such as the "pleasurable deferral of a vacation, the speeding up of a dental appointment, the prolonged storage of a bottle of expensive champagne"

[7] In contrast, Ludvigson (2004) finds a negative correlation between confidence and uncertainty in U.S. data and argues that precautionary saving motives would lead to a positive relationship between consumption growth and lagged uncertainty, which would contradict the observed positive correlation between confidence and consumption growth.

[8] Examples in which time preference shocks can be regarded as a way to capture the relationship between fluctuations in market sentiment and volatility of asset prices, see Barberis, Shleifer, and Vishny (1998) and Dumas, Kurshev, and Uppal (2009).

[9] The authors claim that the presence of habit formation, which implies that lagged consumption growth has predictive power for current consumption growth, might explain the correlation of lagged confidence with current consumption growth as arising from the correlation of lagged confidence with lagged consumption growth.

[10] For a discussion on the origin and the relevance of anticipatory utility, see Frederick, Loewenstein and O'Donoghue (2002).

(p. 666), and defines utility from anticipation as proportional to the future stream of utility from personal consumption, a formalization later borrowed by the few contributions providing asset pricing applications: Caplin and Leahy (2001) investigate the role of anxiety in determining the risk-free rate of return and the equity premium; Kuznitz, Kandel and Fos (2008) study the effect of anticipatory utility on the mean allocation to stocks. Our approach differs from theirs as it considers *external* factors.

The asset pricing literature contains many contributions that, implicitly or explicitly, incorporate state variables. Indeed, Cochrane (2017) argues that virtually every idea behind macro-finance models can be seen as a generalization of the stochastic discount factor obtained by adding a state variable. Our framework explicitly considers a non-separable utility function in consumption and consumer confidence. Early examples of papers worked out in a similar fashion include Eichenbaum, Hansen, and Singleton (1988), Aschauer (1985) and Startz (1989), who let the state variable be leisure, government spending, and the stock of durable goods, respectively.[11] More recently, Piazzesi, Schneider, and Tuzel (2007) introduces housing. In all these papers, the state variable is represented by some *good* other than consumption. Conversely, our approach incorporates traits of psychological nature concerning consumers' time preferences. An example of including an asset demand shifter into an asset pricing model with recursive preferences is Albuquerque, Eichenbaum, Luo and Rebelo (2016). These authors reverse-engineer the properties that a time preference shock should have to replicate some observed stylized facts in the macro-finance literature. We complement their work by investigating whether the intertemporal linkages created by incorporating consumer confidence and habit persistence may act as measurable fundamentals for the asset demand shifter.

Finally, our work relates to models with preferences for early resolution (or recursive utility). At least two fundamental branches of the modern micro-finance literature draw on these models: long-run risks (Bansal and Yaron, 2004; Hansen, Heaton, and Li (2008); Bansal, Kiku, and Yaron, 2012); rare disasters (Rietz, 1988; Barro, 2006, 2009) and persistent-rare disasters (Wachter, 2013). More recently, Andreasen and Jørgensen (2020) disentangle the timing attitude from both the intertemporal elasticity of substitution and the relative risk aversion. The paper closer in spirit to our approach is Melino and Yang (2003). In a framework featuring recursive utility, these authors also introduce a state variable, letting the preference parameters vary across states. In our paper, instead, all preference parameters hold constant and, as such, are not state-contingent.

[11] While these explorations should in principle enhance the performance of the baseline macro-finance approach, at least as long as the newly introduced variables covary positively with consumption growth and the market return, Campbell, Lo, and MacKinlay (1997, p.326) argue that "none of these extra variables greatly improve the ability of the consumption CAPM to fit the data."

2 The model

This section develops a parsimonious macro-finance model with recursive utility incorporating an exogenous state variable. We begin by describing a simple asset pricing framework with a generic state variable, which is possible because the model's derivations are unaffected by the state variable's particular definition (as long as it represents quantities that are beyond the consumer's control). Next, we show that the framework is sufficiently flexible to encompass four different model specifications, which constitute the object of analysis in the next section. Each model specification obtains by defining the state variable in a particular way, with the definition generating the fully-specified model comprising both consumer confidence and year-on-year growth rates, and the remaining specifications then obtained by abstracting from either or both elements, in turn. Then, we offer an illustrative intuition about the model's suitability to replicate the observed financial asset statistics with reasonable preference parameters values. We conclude by stipulating the joint stochastic behavior of consumption and consumer confidence growth rates.

Recursive utility and state variable

Consider a consumption-based asset pricing model in which the consumers' preferences are represented by a recursive utility function à la Kreps and Porteus (1978), with the one-period utility that is non-separable in consumption and an exogenous state variable. Formally, we let the representative consumer's lifetime utility U_t from date t onward be represented by the function

$$U_t = \left[(1-\beta)\left(\kappa_t c_t\right)^{\rho} + \beta \mu_t \left\{U_{t+1}\right\}^{\rho}\right]^{\frac{1}{\rho}} \tag{1}$$

where c is consumption and κ is the state variable. The term $\mu_t \{\cdot\}$ is a 'certainty equivalent' operator, conditional on information at date t, specified as the nonlinear function of the expected value of future lifetime utility

$$\mu_t \left\{U_{t+1}\right\} = \left[E_t \left\{\left(U_{t+1}\right)^{1-\alpha}\right\}\right]^{\frac{1}{1-\alpha}} \tag{2}$$

The preference parameters $\beta > 0$ and $0 < \alpha \neq 1$ represent the subjective discount factor and the relative risk aversion coefficient, respectively; $0 \neq \rho < 1$ governs the intertemporal elasticity of substitution $\eta \equiv 1/\left(1-\rho\right)$.[12]

[12] More precisely, the expression

$$\log U_t = (1-\beta) \log \left\{\kappa_t c_t\right\} + \beta \log \left\{\mu_t \left\{U_{t+1}\right\}\right\}$$

replaces (1) whenever $\rho = 0$.

Under this preference specification, the stochastic discount factor (SDF) is given by

$$m\left\{s_t, s_{t+1}\right\} = \beta\left(x_{t+1}\right)^{\rho-1}\left(\gamma_{t+1}\right)^{\rho}\left(\frac{V\left\{s_{t+1}\right\}}{\mu_{s_t}\left\{V\left\{s_{t+1}\right\}\right\}}\right)^{1-\rho-\alpha} \tag{3}$$

where $x_{t+1} \equiv c_{t+1}/c_t$ and $\gamma_{t+1} \equiv \kappa_{t+1}/\kappa_t$ are respectively the consumption and the state variable growth factors, $V\{\cdot\}$ is the value function in equilibrium, and $s = (\kappa, \gamma, c, x)$ denotes the aggregate state.[13]

The SDF incorporates three terms. The first term, $\beta\left(x_{t+1}\right)^{\rho-1}$, is the product between the subjective discount factor and a non-increasing power function of consumption growth. It represents the SDF in the seminal contribution by Mehra and Prescott (1985) and is one of the two terms comprising the SDF in the EZW model. The second term, $\left(\gamma_{t+1}\right)^{\rho}$, is a concave function of the innovation in the state variable. Taken in isolation, it reflects the impact of the state variable on the representative consumer's choice abstracting from uncertainty. The third term, $\left(V\left\{s_{t+1}\right\}/\mu_{s_t}\left\{V\left\{s_{t+1}\right\}\right\}\right)^{1-\rho-\alpha}$, involves the representative consumer's value function and reflects the consumer's preferences for the timing of resolution of uncertainty. If early resolution is preferred, i.e., $1-\rho-\alpha < 0$, then asset payoffs in states where realized lifetime utility is lower than the conditional certainty equivalent will have a greater impact on the asset price than payoffs in states where the opposite occurs, just like in the EZW model. Of course, the difference is that here the value function also depends on the state variable: that is, the state variable affects the magnitude of the potential rise in the volatility of the SDF relative to that generated by the first, standard, term.[14]

State variable, consumer confidence and year-on-year growth rates

We may specialize the model by giving the state variable an explicit definition. Specifically, we wish to generate four different model specifications, each identified by a capital letter. Model A incorporates both consumer confidence and habit persistence. Models B and C abstract from habit persistence and consumer confidence, respectively. Model D disregards both elements. Each state variable definition identifies a different SDF, which we will use to perform our quantitative analysis in the next section. To illustrate the link between habit persistence and year-on-year convention in a transparent fashion, it proves convenient to state the length of a model's period explicitly: in line with our quantitative analysis, we let a quarter represent the time elapsing between the dates t and $t+1$.

[13] See Appendix A.1 for a formal derivation of equation (3).

[14] If the consumer is indifferent to the timing of resolution of uncertainty, i.e. $1-\rho-\alpha = 0$, then the SDF is ordinally equivalent to $m\left\{s_t, s_{t+1}\right\} = \beta\left(x_{t+1}\right)^{-\alpha}\left(\gamma_{t+1}\right)^{1-\alpha}$. In this case, the term $\left(\gamma_{t+1}\right)^{1-\alpha}$ captures the response of consumer choice to uncertainty: payoffs in states where the state variable is above average have a smaller impact than payoffs in states where the opposite occurs if the coefficient of relative risk aversion is larger than one, and vice versa.

We begin with the simplest case, i.e., model D, which abstracts from the state variable altogether. Let $\kappa_t^D = 1$, for all dates t. It immediately follows that $\gamma_{t+1}^D = 1$, and the SDF reduces to

$$m^D\{s_t, s_{t+1}\} = \beta (x_{t+1})^{\rho-1} \left(\frac{V^D\{s_{t+1}\}}{\mu_{s_t}\{V^D\{s_{t+1}\}\}} \right)^{1-\rho-\alpha} \tag{4}$$

Equation (4) corresponds to the SDF of the EZW model.

We now introduce external habit persistence into the framework while still abstracting from consumer confidence. This setting corresponds to model C. Let $\kappa_t^C = \xi_t$, where

$$\xi_t \equiv (c_{t-1} \cdot c_{t-2} \cdot c_{t-3})^{(\rho-1)/\rho} \tag{5}$$

is a composite function defined over three lagged values of consumption. The state variable growth rate is $\gamma_{t+1}^C = (c_t/c_{t-3})^{(\rho-1)/\rho}$. Plugging this value into (3), and denoting $\tilde{x}_{t+1} \equiv c_{t+1}/c_{t-3}$, yields

$$m^C\{s_t, s_{t+1}\} = \beta (\tilde{x}_{t+1})^{\rho-1} \left(\frac{V^C\{s_{t+1}\}}{\mu_{s_t}\{V^C\{s_{t+1}\}\}} \right)^{1-\rho-\alpha} \tag{6}$$

Comparing (6) with (4), we may notice that the growth rate of consumption is now computed over four quarters (hence, using the year-on-year convention).

Next, we incorporate consumer confidence and disregard habit persistence, which specifies model B. Let $\kappa_t^B = \psi_t$, where ψ_t denotes the value of consumer confidence at date t. We have $\gamma_{t+1}^B \equiv \psi_{t+1}/\psi_t$ from which, in conjunction with (3) and letting $\theta_{t+1} \equiv \psi_{t+1}/\psi_t$, we obtain

$$m^B\{s_t, s_{t+1}\} = \beta (x_{t+1})^{\rho-1} (\theta_{t+1})^{\rho} \left(\frac{V^B\{s_{t+1}\}}{\mu_{s_t}\{V^B\{s_{t+1}\}\}} \right)^{1-\rho-\alpha} \tag{7}$$

Under this specification, the SDF explicitly features the consumer confidence growth factor as an exogenous state variable.

Finally, we simultaneously consider consumer confidence and habit persistence, recreating the specification of model A. Let $\kappa_t^A \equiv \psi_t \cdot \varphi_t \cdot \xi_t$, where

$$\varphi_t \equiv \psi_{t-1} \cdot \psi_{t-2} \cdot \psi_{t-3} \tag{8}$$

is a composite function defined over three lagged values of consumer confidence, and ξ_t is given by (5) as before. The state variable growth rate becomes

$$\gamma_{t+1}^A = \frac{\psi_{t+1}}{\psi_{t-3}} \left(\frac{c_t}{c_{t-3}} \right)^{(\rho-1)/\rho}$$

and, denoting $\tilde{\theta}_{t+1} \equiv \psi_{t+1}/\psi_{t-3}$, the resulting SDF reads

$$m^A\left\{s_t, s_{t+1}\right\} = \beta\left(\tilde{x}_{t+1}\right)^{\rho-1}\left(\tilde{\theta}_{t+1}\right)^{\rho}\left(\frac{V^A\left\{s_{t+1}\right\}}{\mu_{s_t}\left\{V^A\left\{s_{t+1}\right\}\right\}}\right)^{1-\rho-\alpha} \tag{9}$$

This expression corresponds to the price kernel under the fully specified approach. We may note that the state variable explicitly incorporates the consumer confidence growth rate in the SDF *and* entails year-on-year growth rate computations. In the next section, (9) will also identify the SDF for model E. What distinguishes models A and E is that they use different indicators to measure consumer confidence.

Consumer confidence, year-on-year growth rates and the SDF

The different versions of the SDF depicted by (4), (6), (7) and (9) lead to different asset price moments. In order to illustrate why simultaneously incorporating consumer confidence and habit persistence may help in replicating the observed asset pricing behavior, we graphically compare the simulations of the first two moments of the SDF generated by (4) and (9). We then use some basic financial relations to guide our reasoning and develop our intuition.

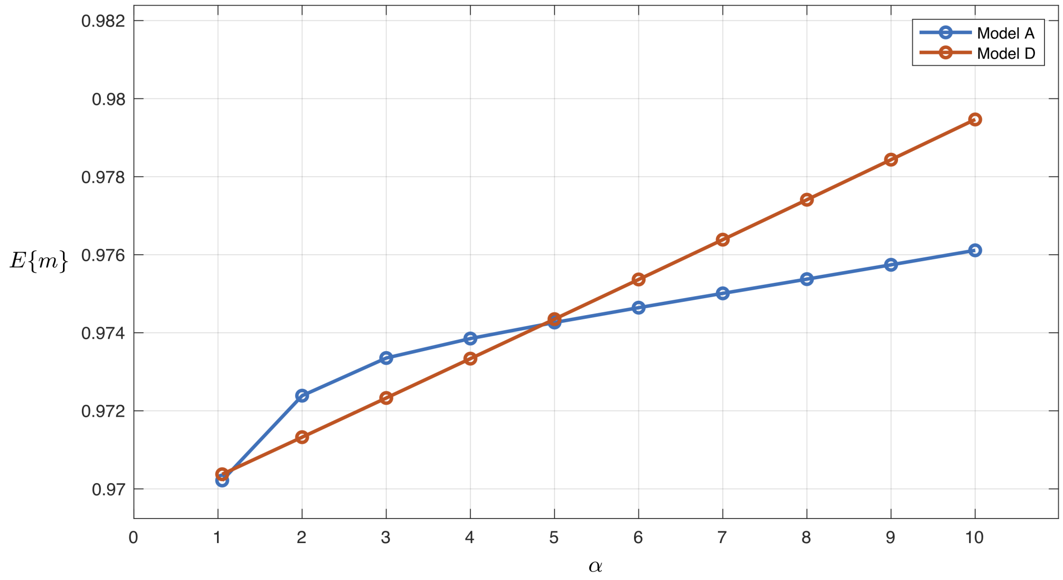
Figure 1 illustrates the evolution of mean and standard deviation of the SDF as the relative risk aversion coefficient varies. We let the value of the subjective discount factor be $\beta = 0.99$ and set the intertemporal elasticity of substitution to one ($\rho = 0$). The values of the annualized moments of consumption growth and consumer confidence innovation, as well as those of the transitional probabilities, are computed using the dataset and the techniques described in Section 3. The top panel deals with the expected value of the SDF. We note that the values delivered by our approach (model A) are, at low levels of RRA, larger than those obtained by the EZW framework (model D). The bottom panel concerns the volatility of the SDF. There, the values delivered by model A are substantially higher than model D's at low levels of RRA; the gap narrows as the RRA coefficient rises, yet the magnitude of SDF generated by our approach remains significantly larger than the EZW's.

In order to get a quick grasp at how the SDF generates asset prices and the resulting returns, consider the following illustrative exercise. Recall that $cov\left\{m, R\right\} = E\left\{m\,R\right\} - E\left\{m\right\}E\left\{R\right\}$ and $\rho_{m,R} = cov\left\{m, R\right\}/\left(\sigma\left\{m\right\}\sigma\left\{R\right\}\right)$; furthermore, consider that for any asset on the efficient mean-variance frontier it holds that $R \approx a - b\,m$, with a, b some positive numbers, and therefore $\rho_{m,R} = -1$.[15] Then, from the central asset pricing formula, $E\left\{m\,R\right\} = 1$,

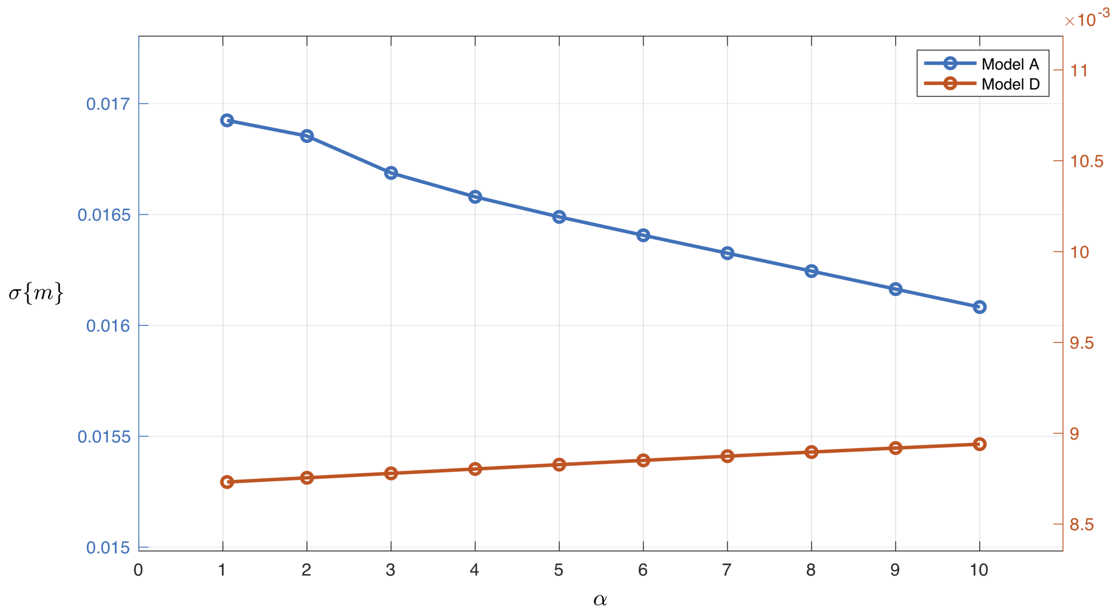
[15] More precisely, for $R \approx a - b\,m$ to hold, the risky asset should be a good approximation of the market portfolio, and the financial market should not be too far from being complete. For a more exhaustive discussion, see, e.g., Cochrane (2005, Chapter 1).

Figure 1. SDF mean and volatility as the relative risk aversion coefficient varies.

Panel I. Stochastic discount factor expected value.



Panel II. Stochastic discount factor standard deviation.



The figure illustrates the patterns of the stochastic discount factor mean (Panel I) and standard deviation (Panel II) as the relative risk aversion coefficient varies for two models: model A incorporates both consumer confidence and habit persistence; model D neither. The subjective discount factor is set to $\beta = 0.99$, and the intertemporal elasticity of substitution to one ($\rho = 0$). The transitional probabilities are calibrated as described in Section 3. The SDF standard deviations reported in Panel II are expressed in different scales: the left one refers to model A, the right one to model D.

we may obtain the following three equations that our illustrative simulation must obey

$$E\left\{R^f\right\} \;=\; 1/E\{m\} \tag{10}$$

$$E\left\{R_m - R^f\right\} \;\approx\; \frac{b\,\sigma^2\{m\}}{E\{m\}} \tag{11}$$

$$\sigma\{R_m\} \;\approx\; b\,\sigma\{m\} \tag{12}$$

where $E\left\{R^f\right\}$ and $E\left\{R_m - R^f\right\}$ are the annualized risk-free rate and the equity premium unconditional means, $\sigma\{R_m\}$ is the annualized market return unconditional volatility and b is a value governed by the preference parameters. From (10), we learn that the expected risk-free return is merely the reciprocal of the SDF expected value. Thus, our exercise suggests that incorporating consumer confidence and using the year-on-year convention can predict lower riskless rates than a framework abstracting from them for modest levels of risk aversion. From (11), we establish that the equity premium is proportional to the ratio between the SDF's variance and mean. In light of our simulation results, we expect model A to predict larger equity premia at virtually any level of relative risk aversion. Finally, (12) indicates that the equity return volatility is proportional to the SDF's standard deviation. Our simulations are then suggestive of the predictions on $\sigma\{R_m\}$ following a similar pattern as those on $E\left\{R_m - R^f\right\}$. Each of these three predictions has the potential to represent an improvement over those delivered by the EZW model.

Dynamics of consumption and consumer confidence growth rates

We model the joint process for consumption growth x and consumer confidence innovation θ as the first-order autoregressive scheme

$$y_t = g + Ay_{t-1} + \varepsilon_t \tag{13}$$

where $y_t \equiv \left[x_t - \bar{x}, \theta_t - \bar{\theta}\right]$ is a 2×1 vector collecting the detrended growth factors, A is a 2×2 matrix containing the autoregression coefficients, and ε_t is a 2×1 vector white noise process. It is assumed that the elements ε_{it} of ε_t are mutually independent with probability $\Pr\{\varepsilon_{it} \leq u\} = Z_i\{u/\sigma(\varepsilon_i)\}$, where Z_i is a standardized Gaussian distribution.

We approximate (13) with a finite-state Markov chain using Tauchen's (1986) method. The method consists of choosing values of the variables and the transition probabilities for each state so that the resulting discrete Markov chain mimics the underlying continuous-valued autoregression closely. It relies on the well-established Markov chain suitability to adequately capture the relevant time series' statistical properties (after an adjustment for trend). The probability of each state is determined by computing the cumulative density for a finite interval of the distributional domain, around the values that the two variables take in that particular state. The resulting probabilities comprise the so-called transitional matrix of the Markov chain. By construction, this probability distribution simultaneously accounts for each variable's volatility and autocorrelation, along with the cross-correlation between the two variables.

3 Quantitative analysis

We now turn to illustrate the model outcomes. We critically compare our results with those reported by Bansal and Yaron (2004) regarding the long-run risks (BY) model and those obtained by the EZW model, here a special case of our approach. We begin by describing the data that we use to calibrate the Markov chain governing the model's stochastic process, along with those that we use as targets to calibrate the preference parameters and assess the model's predictions. We then explain the calibration procedure and illustrate and discuss the model outcomes. Finally, we report on several exercises that we carry out to evaluate the robustness of our findings.

Data

We need to feed the model data on consumption growth and consumer confidence innovation to obtain predictions regarding the risk-free rate, the market return, and the price-dividend ratio. Naturally, we also need data on the latter variables to create targets for calibrating the model and assessing its performance. We detail our sources in turn.[16] Our database spans from the third quarter of 1967 to the last quarter of 2018, thereby containing 206 observations. Growth factors are computed using the year-on-year convention, as well as the more customary (to the macro-finance literature) quarter-on-quarter convention.

The consumption growth time series is calculated using the U.S. Bureau of Economic Analysis' data. The United States personal consumption expenditures on non-durable goods and services, expressed in nominal seasonally adjusted annual rates, are deflated using the seasonally adjusted United States personal consumption expenditures 2012 year-base chain-type price index. The resulting monthly figures are converted in per-capita terms using the United States population. We then average the data at a quarterly frequency.

The consumer confidence innovation's time series is calculated using the Conference Board's Consumer Confidence Index (CC) monthly data, retrieved from the Macrobond Financial database.[17] The index is based on a five-question survey, which includes queries about current and future general market conditions and job availability. Specifically, the questions seek the respondents' appraisal regarding current (i) business conditions and (ii) employment conditions;

[16] Unless otherwise specified, the time series are sourced at a monthly frequency from the Federal Reserve Economic Data, available at the webpage: `https://fred.stlouisfed.org`.

[17] For further information, visit the webpage: `macrobond.com`.

and the respondents' expectations six months hence regarding (iii) business conditions, (iv) employment conditions, and (v) their total family income. Each question can be given a positive, negative, or neutral answer. The answers' resulting proportions are seasonally adjusted. For each question, the proportion of positive answers is divided by the sum of the proportions of positive and negative answers to obtain an indicator, which is then standardized using the average indicator of the calendar year 1985 to calculate the index level. The overall index value is calculated as the simple monthly average of the five questions' index levels.[18] The index values are then averaged at a quarterly frequency.

The market return time series is derived from the price and dividend time series of the Standard & Poor's 500 composite index, sourced monthly from Shiller's database.[19] The risk-free rate is calculated using the three-month Treasury bill secondary market rate. Treasury bills rates, market prices, and dividends are expressed in real terms through the same price index used to deflate consumption growth data. In order to aggregate the data at a quarterly frequency, dividends are cumulated over the relevant three months; Treasury bills rates are capitalized over the same period. The market price corresponds to the last month's observation of the quarter. The market return is computed as the sum of the current price and dividends divided by the lagged price.

In one of our robustness exercises, we also use the University of Michigan's Consumer Sentiment Index, sourced from the Macrobond Financial database, as an alternative measure of consumer confidence. The index is constructed similarly to the Conference Board's Consumer Confidence Index, although the sample design and the index estimation are substantially different.[20] This indicator is averaged quarterly over the period covered by our database, too.

Calibration

We need to calibrate two sets of objects to allow the model to deliver the simulated unconditional means and standard deviations of the risk-free rate, the market return, and the price-dividend ratio: the transitional probabilities and the preference parameters governing the consumer's subjective time discounting, relative risk aversion and intertemporal elasticity of substitution. The transitional probability distribution is a prerequisite to run our simulation, so we deal with it first. Once the probabilities are calculated, we run an iterative procedure to identify the preferences parameters.

As we explained in the last paragraph of the last section, we use Tauchen's (1986) method to derive the Markov chain probabilities from a continuous-valued stochastic process.[21] The method

[18] Additional details can be found in the Consumer Confidence Survey Technical note, available at the webpage: `conference-board.org/pdf_free/press/TechnicalPDF_4134_1298367128.pdf`.

[19] Shiller's database is available at the webpage: `econ.yale.edu/~shiller/data/ie_data.xls`.

[20] For more information about the Consumer Sentiment Index, visit the webpage: `sca.isr.umich.edu`.

[21] The method is formally discussed in Appendix A.2.

consists of estimating the autoregressive scheme (13). The coefficients of the autoregression, A, and the volatilities of the error terms, ε_t, are used to compute the variance-covariance matrix of consumption growth and consumer confidence, Σ_y. The elements of Σ_y are then used to produce the values, $\bar{y}_s^v$, that the variables v take in each state s, as well as the relevant Markov chain transitional probabilities, $\pi(s, s')$, from state s to state s'. For each variable, the state-specific values $\bar{y}_s^v$ are equidistant deviations from the variable mean in both directions, with the broader deviation representing the largest shock the variable is allowed to take in the Markov chain. The probabilities associated with the states are the cumulative density of regularly spaced intervals of the joint distributional domain, around the values that the two variables take in each given state.

The number of states and the magnitudes of the largest shocks must be determined ex-ante. In our benchmark exercise, we assign five states ($n = 5$) to each variable and set the largest shock to be equal to three times ($q = 3$) the magnitude of the relevant standard deviation. In order to assess how our choices affect the model's outcomes, we also consider a nine-state case ($n = 9$), as used by Tauchen (1986) in his original contribution, and a two-state case ($n = 2$), the customary choice in the literature. For the same reason, we run robustness exercises regarding the magnitude of the largest shock by considering the typical one-standard-deviation case ($q = 1$).

The procedure to determine the three preference parameters is as follows. We search for values of the parameter that minimize a constrained quadratic loss function. The constraints are chosen to reflect the parameter values admitted by the existing contributions in the literature. Specifically, the subjective discount factor can take values no larger than one, i.e., $\beta \in (0.9, 1)$. The relative risk aversion coefficient is assumed to be positive but no larger than 10; the upper bound considered reasonable by Mehra and Prescott (1985), i.e., $\alpha \in (0, 10)$. The intertemporal elasticity of substitution (IES) is, as always, lower-bounded in zero. Whether the magnitude of IES may or may not be greater than one is a source of considerable debate.[22] On the one hand, Hall (1988) famously estimates IES to be well below one (around 0.1). On the other hand, a value above one is consistent with the findings of several contributions in the literature since Hansen and Singleton (1982). Furthermore, Bansal and Yaron (2004) show that an above-unity intertemporal elasticity of substitution is essential for rationalizing the observed correlation between consumption volatility and price-dividend ratios. In light of this evidence, we choose $\eta \in (0, 2)$ to constrain the minimization problem concerning the intertemporal elasticity of substitution.

The quadratic loss function is given by the sum of squares of the deviations of the simulated values of three targets (one per parameter) from the observed ones. In our benchmark exercise, the data targets are: (i) the mean of the risk-free rate, $E\{R_f\}$, to pinpoint the subjective

[22] For a review of the empirical literature on the intertemporal elasticity of substitution, see Thimme (2017).

discount factor, β; (ii) the mean of the market return, $E\{R_m\}$, to pinpoint the relative risk aversion coefficient, α; (iii) the standard deviation of the risk-free rate, $\sigma\{R_f\}$, to pinpoint the intertemporal elasticity of substitution, η. In order to evaluate our choice of targets, we also consider some alternative specifications by replacing (iii), in turn, with the standard deviation of the market return, $\sigma\{R_m\}$, the mean, $E\{\Omega_m\}$, and the standard deviation, $\sigma\{\omega_m\}$, of the (log) price-dividend ratio.

Results

Table 1 illustrates the outcomes of the model obtained by using our benchmark calibration. As discussed in the previous subsection, for both consumption growth and consumer confidence innovation, the number of states is five ($n = 5$), and the largest shock is three times the standard deviation ($q = 3$). The data targets for the subjective discount factor, β, the intertemporal elasticity of substitution, η, and the relative risk aversion coefficient, α, are the mean, $E\{R_f\}$, and the standard deviation of the risk-free rate, $\sigma\{R_f\}$, and the mean of the market return, $E\{R_m\}$, respectively.

Each row of the table refers to a particular model, coded by a distinct capital letter. The five letters refer to different specifications of our framework. We offer more details below, as the discussion will focus on each specification in turn (to be precise, model E is discussed in the next subsection). Regarding the last model (BY), the table reports the original findings by Bansal and Yaron (2004). For each model, the columns of the table report the values of the calibrated parameters (β, η and α), the targeted statistics ($E\{R_f\}$, $E\{R_m\}$ and $\sigma\{R_f\}$), the untargeted statistics ($\sigma\{R_m\}$, $E\{\Omega_m\}$ and $\sigma\{\omega_m\}$) and the equity premium ($E\{R_m - R_f\}$). In order to aid visual comparison, the table also reports the ratio of the simulated statistics to the observed ones (in percentage terms). For the BY model, the statistics are not reported: since Bansal and Yaron (2004) use a different period (1929-1988), the figures they report are not directly comparable to the ones produced by our models.

Model A refers to the full specification of our approach. The relevant stochastic discount factor (SDF), expressed by (9), includes consumer confidence as a state variable in the representative consumer's preference specification, and all moments are based on growth rates computed using the year-on-year convention. In line with our discussion in the previous section, we will refer to model A when simultaneously considering consumer confidence and habit formation. The examination of the relevant rows of Table 1 reveals that the model performs well in replicating the targeted statistics, whereas it does not do an excellent job in reproducing the untargeted ones. Importantly, these results of Model A obtain with very reasonable values of the calibrated parameters.

Table 1. Benchmark calibration.

Model	β	η	α	$E\{R_m - R_f\}$	$E\{R_f\}$	$E\{R_m\}$	$\sigma\{R_f\}$	$\sigma\{R_m\}$	$E\{\Omega_m\}$	$\sigma\{\omega_m\}$
Panel I. Calibrated models and simulated statistics.										
	Calibrated parameters				Targeted statistics			Untargeted statistics		
A	0.9951	1.066	5.72	0.0641	0.0118	0.0758	0.0227	0.0156	18.89	0.0504
				100%	100%	100%	99%	10%	46%	12%
B	0.9990	1.989	9.49	0.0304	0.0340	0.0644	0.0084	0.0301	25.15	0.0469
				47%	289%	85%	37%	18%	62%	11%
C	0.9665	2.000	10.00	0.0305	0.0264	0.0569	0.0066	0.0101	29.47	0.0913
				48%	225%	75%	29%	6%	72%	22%
D	0.9710	1.224	9.99	0.0122	0.0376	0.0498	0.0078	0.0122	37.15	0.0300
				19%	320%	66%	34%	7%	91%	7%
E	0.9914	1.175	6.26	0.0625	0.0125	0.0750	0.0256	0.0158	19.31	0.0584
				98%	106%	99%	112%	10%	47%	14%
Panel II. Comparative model and observed statistics.										
BY	0.9980	1.500	10.00	108%	108%	108%	59%	96%	75%	72%
Observed statistics				0.0640	0.0118	0.0758	0.0229	0.1640	40.83	0.4203

The table reports the results obtained using five different models specifications, each identified by a capital letter: A incorporates consumer confidence and habit persistence; B only consumer confidence; C only habit persistence; D neither (so it is a version of the EZW model); E incorporates consumer sentiment and habit persistence. The expressions β, η and α respectively refer to the subjective discount factor, the intertemporal elasticity of substitution and the relative risk aversion coefficient; $E\{R_m – R_f\}$ is the equity premium; $E\{R_f\}$, $E\{R_m\}$ and $E\{\Omega_m\}$ are, respectively, the risk-free rate, the market return and the price-dividend ratio means; $\sigma\{R_f\}$, $\sigma\{R_m\}$ and $\sigma\{\omega_m\}$ are the corresponding volatilities. In Panel I, all entries are based on the relevant model's calibration of the three preference parameters, using the three statistics indicated in the table as targets, and of the transitional probabilities. Panel II reports the parameter values used, and the results obtained, by Bansal and Yaron (2004) as well as the observed values of the relevant statistics.

The conclusions we draw from the results delivered by model A are confirmed when compared with the simulated statistics offered by Bansal and Yaron (2004). In particular, model A compares favorably with model BY regarding the targeted statistics, especially the one concerning the standard deviation of the risk-free rate. The discrepancies between the observed statistics and the ones simulated by model A are within a 1% deviation. In contrast, those arising with respect to model BY range from 8% for $E\{R_f\}$ and $E\{R_m\}$ to 41% for $\sigma\{R_f\}$. Model A's performance deteriorates regarding the untargeted statistics, particularly those concerning the volatilities of the mean return and price-dividend ratio. The differences between the observed statistics and the ones simulated by model A range from over 50% for $E\{\Omega_m\}$ to nearly 90% for $\sigma\{R_m\}$ and $\sigma\{\omega_m\}$. Conversely, those arising for model BY vary from 4% for $\sigma\{R_m\}$ to less than 30% for $E\{\Omega_m\}$ and $\sigma\{\omega_m\}$. It is also worth noting that model A's results are obtained with calibrated preference parameters that are farther apart from their upper bounds than the ones imposed by Bansal and Yaron (2004) to their model.

Model B features consumer confidence as a state variable, but it abstracts from habit formation using the more standard quarter-to-quarter convention to compute time-variations. The relevant SDF is represented by (7). From Table 1, we learn that model B's performance drops significantly relative to model A's. The calibrated parameters η and α are close to the relevant upper bounds. Notwithstanding, all targeted statistics are far off the data targets: compared to their observed counterparts, the simulated $E\{R_f\}$ is almost three times as large, and $\sigma\{R_f\}$ is more than 60% lower; the best-targeted statistics is $E\{R_m\}$, whose simulated value nonetheless deviates 15% below the observed one. The model records a slight improvement in terms of untargeted statistics: the differences between the simulated and observed figures of $\sigma\{R_m\}$ and $E\{\Omega_m\}$ are respectively 8% and 16% smaller than those produced by model A, whereas those of $\sigma\{\omega_m\}$ are similar. Model BY outperforms model B across the whole board.

Model C represents model B's mirroring specification: it excludes consumer confidence from consumer preferences but considers habit formation through the year-on-year calculation of time-variations. The relevant SDF is given by (6). Table 1 shows that, also under this specification, the model generally underperforms relative to model A, both in terms of calibrated parameters and targeted and untargeted statistics. The only improvements concern the moments of the price-dividend ratio, whose discrepancies between simulated and observed values reduce by 26% for $E\{\Omega_m\}$ and 10% for $\sigma\{\omega_m\}$. Once again, the model is dominated by BY in terms of every statistic.

Jointly considered, the outcomes of models B and C suggest a significant role for both consumer confidence and habit formation. This implication finds further support in the results obtained by simulating model D, which neglects the role of consumer confidence as a state variable and makes use of time-variations computed using the quarter-on-quarter convention. In short, Model D corresponds to a version of the original (EZW) model by Epstein and Zin

(1989) and Weil (1989), whose SDF is depicted by (4). While the calibration of η represents an improvement relative to models B and C, the remaining figures indicate a further deterioration in the model's performance with the exception, once again, of the statistics $E\{\Omega_m\}$, which deviates from its observed counterpart by less than 10%.

Robustness

We check the sensitivity of our results along three dimensions. The first involves an alternative measure for consumer confidence. The second considers the use of different targets to calibrate the three preference parameters. The third examines alternative parameter specifications for calibrating the transitional probability matrix. In this subsection, we present our findings for each set of relevant exercises in turn.

The Conference Board's Consumer Confidence Index is often considered jointly with the University of Michigan's Consumer Sentiment Index.[23] Therefore, it seems natural to explore whether the results of the model extend to using the Sentiment Index as a proxy for consumer confidence. The outcomes of calibrating the model on consumption growth and consumer sentiment are reported in Table 1 under model E. The model is otherwise specified as the benchmark model A (that is, the state variable is involved in the analysis, and we consider year-on-year growth rates). The table reveals that the simulation of model E delivers similar figures as model A. To a modest deterioration in replicating the targeted statistics corresponds a slight improvement in matching the untargeted ones. We can then conclude that the same assessment of model A also applies here.

As discussed in the previous subsections, our exercises involve a total of six statistics, three of which are used as targets to calibrate the subjective discount factor, β, the relative risk aversion coefficient, α, and the intertemporal elasticity of substitution, η. In the benchmark exercise, we targeted the statistics that we deem more informative for each preference parameter, namely the mean of the risk-free rate, $E\{R_f\}$, the mean of the market return, $E\{R_m\}$, and the risk-free rate standard deviation, $\sigma\{R_f\}$, respectively. In order to assess the merit of our choice, Table 2 reports the results obtained by replacing the third target with (panel I) the market return standard deviation, $\sigma\{R_m\}$, (panel II) the mean of the price-dividend ratio, $E\{\Omega_m\}$, and (panel III) the (log) price-dividend ratio standard deviation, $\sigma\{\omega_m\}$, in turn. For brevity, we restrict our attention to the fully specified model (A) and the EZW model (D).

From Table 2, we learn that model A is sensitive to the choice of the targets. The calibrated η jumps to its upper-bound in panel I, while it remains close to one in panels II and III; α is lower than the benchmark case in panels I and II, then higher in panel III. These sizeable variations

[23] For a discussion of the historical reasons for this pairing, together with a detailed description of differences and similarities between the two indices, see Bram and Ludvigson (1998).

Table 2. Calibration with alternative targets.

Panel I. Targeting the volatility of the market return.

Model	Calibrated parameters				Targeted statistics			Untargeted statistics		
	β	η	α	$E\{R_m - R_f\}$	$E\{R_f\}$	$E\{R_m\}$	$\sigma\{R_m\}$	$\sigma\{R_f\}$	$E\{\Omega_m\}$	$\sigma\{\omega_m\}$
A	0.9720	2.000	1.01	0.0842	0.0045	0.0887	0.1126	0.0974	54.21	0.4511
				132%	38%	117%	69%	426%	133%	107%
D	0.9653	2.000	10.00	0.0140	0.0377	0.0517	0.0153	0.0048	35.14	0.0342
				22%	320%	68%	9%	21%	86%	8%
BY	0.9980	1.500	10.00	108%	108%	108%	96%	59%	75%	72%
	Observed statistics			0.0640	0.0118	0.0758	0.1640	0.0229	40.83	0.4203

Panel II. Targeting the price-dividend ratio.

Model	Calibrated parameters				Targeted statistics			Untargeted statistics		
	β	η	α	$E\{R_m - R_f\}$	$E\{R_f\}$	$E\{R_m\}$	$E\{\Omega_m\}$	$\sigma\{R_f\}$	$\sigma\{R_m\}$	$\sigma\{\omega_m\}$
A	0.9890	1.080	3.09	0.0679	0.0079	0.0758	19.15	0.0198	0.0188	0.0742
				106%	67%	100%	47%	87%	11%	18%
D	0.9680	1.996	10.00	0.0140	0.0348	0.0488	39.04	0.0048	0.0152	0.0342
				22%	296%	64%	96%	21%	9%	8%
BY	0.9980	1.500	10.00	108%	108%	108%	75%	59%	96%	72%
	Observed statistics			0.0640	0.0118	0.0758	40.83	0.0229	0.1640	0.4203

Panel III. Targeting the volatility of the price-dividend ratio.

Model	Calibrated parameters				Targeted statistics			Untargeted statistics		
	β	η	α	$E\{R_m - R_f\}$	$E\{R_f\}$	$E\{R_m\}$	$\sigma\{\omega_m\}$	$\sigma\{R_f\}$	$\sigma\{R_m\}$	$E\{\Omega_m\}$
A	0.9988	1.071	6.67	0.0661	0.0110	0.0771	0.0493	0.0240	0.0155	18.45
				103%	94%	102%	12%	105%	9%	45%
D	0.9663	1.999	10.00	0.0140	0.0366	0.0506	0.0342	0.0048	0.0153	36.46
				22%	311%	67%	8%	21%	9%	89%
BY	0.9980	1.500	10.00	108%	108%	108%	72%	59%	96%	75%
	Observed statistics			0.0640	0.0118	0.0758	0.4203	0.0229	0.1640	40.83

The table reports the results obtained using three different calibration strategies (Panels I-III) for two models specifications, each identified by a capital letter: A incorporates consumer confidence and habit persistence; D neither (so it is a version of the EZW model). The expressions β, η and α respectively refer to the subjective discount factor, the intertemporal elasticity of substitution and the relative risk aversion coefficient; $E\{R_m - R_f\}$ is the equity premium; $E\{R_f\}$, $E\{R_m\}$ and $E\{\Omega_m\}$ are, respectively, the risk-free rate, the market return and the price-dividend ratio means; $\sigma\{R_f\}$, $\sigma\{R_m\}$ and $\sigma\{\omega_m\}$ are the corresponding volatilities. Each Panel corresponds to a calibration obtained by replacing the target $\sigma\{R_f\}$ with $\sigma\{R_m\}$, $E\{\Omega_m\}$ and $\sigma\{\omega_m\}$, in turn. Each Panel also reports the parameter values used, and the results obtained, by Bansal and Yaron (2004) as well as the observed values of the relevant statistics.

Table 3. Calibration with alternative transitional probabilities.

Model	β	η	α	$E\{R_m - R_f\}$	$E\{R_f\}$	$E\{R_m\}$	$\sigma\{R_f\}$	$\sigma\{R_m\}$	$E\{\Omega_m\}$	$\sigma\{\omega_m\}$
Panel I. Nine states (n = 9), three standard deviations (q = 3).										
	Calibrated parameters				Targeted statistics			Untargeted statistics		
A	0.9989	1.073	8.33	0.0646	0.0116	0.0762	0.0215	0.0124	18.70	0.0429
				101%	99%	101%	94%	8%	46%	10%
D	0.9735	1.053	9.99	0.0100	0.0388	0.0487	0.0084	0.0102	38.32	0.0263
				16%	329%	64%	37%	6%	94%	6%
Panel II. Five states (n = 5), one standard deviation (q = 1).										
	Calibrated parameters				Targeted statistics			Untargeted statistics		
A	0.9990	1.570	8.21	0.0461	0.0213	0.0673	0.0336	0.0184	22.53	0.0516
				72%	181%	89%	147%	11%	55%	12%
D	0.9988	0.448	9.98	0.0030	0.0428	0.0458	0.0097	0.0052	41.84	0.0118
				5%	364%	60%	42%	3%	102%	3%
Panel III. Two states (n = 2), one standard deviation (q = 1).										
	Calibrated parameters				Targeted statistics			Untargeted statistics		
A	0.9990	1.322	6.23	0.0415	0.0236	0.0651	0.0342	0.0148	23.48	0.0588
				65%	201%	86%	149%	9%	58%	14%
D	0.9848	0.622	9.99	0.0069	0.0404	0.0472	0.0107	0.0083	40.12	0.0199
				11%	343%	62%	47%	5%	98%	5%
Panel IV. Comparative model and observed statistics.										
BY	0.9980	1.500	10.00	108%	108%	108%	59%	96%	75%	72%
	Observed statistics			0.0640	0.0118	0.0758	0.0229	0.1640	40.83	0.4203

The table reports the results obtained using three different calibration strategies (Panels I-III) for two models specifications, each identified by a capital letter: A incorporates consumer confidence and habit persistence; D neither (so it is a version of the EZW model). The expressions β, η and α respectively refer to the subjective discount factor, the intertemporal elasticity of substitution and the relative risk aversion coefficient; $E\{R_m - R_f\}$ is the equity premium; $E\{R_f\}$, $E\{R_m\}$ and $E\{\Omega_m\}$ are, respectively, the risk-free rate, the market return and the price-dividend ratio means; $\sigma\{R_f\}$, $\sigma\{R_m\}$ and $\sigma\{\omega_m\}$ are the corresponding volatilities. Each Panel corresponds to a calibration obtained by modifying the number of states and/or the value of the largest shock that each variable can take. Panel IV reports the parameter values used, and the results obtained, by Bansal and Yaron (2004) as well as the observed values of the relevant statistics.

cause several sways in the simulated statistics. In panel I, the improvement in replicating $\sigma\{R_m\}$ is accompanied by the one in matching $E\{\Omega_m\}$ and $\sigma\{\omega_m\}$; there is, however, a striking overshooting of $\sigma\{R_f\}$ and a substantial drop in $E\{R_f\}$. Panel II reports figures that are more in line with those delivered by the benchmark exercise, though we register a deterioration in replicating $E\{R_f\}$ and $\sigma\{R_f\}$ together with a limited improvement regarding $\sigma\{\omega_m\}$. In panel III, there is a slight deterioration in replicating all statistics. Regarding the EZW model, η and α are in the neighborhood of the respective upper bounds in all panels. Therefore, the simulated figures are quite similar and generally indicate slightly better performance, except for $\sigma\{R_f\}$. Overall, using targets alternative to those chosen initially worsens the model's outcomes.

The benchmark exercise is based on a Markov chain featuring five states of each variable ($n = 5$, for a total of 25 states) and three times each standard deviations as the largest shocks ($q = 3$). Table 3 portrays the outcomes of models A and D when we let these two figures vary. Specifically, in panel I, the number of states rises to nine ($n = 9$), as originally proposed by Tauchen (1986); in panel II, the largest shock decreases to one standard deviation ($q = 1$); in panel III, the number of states drops to two ($n = 2$) in conjunction with the largest shock falling to one standard deviation, as in Mehra and Prescott (1985) and many contributions ever since.

The table shows a slight to limited deterioration in the model's ability to replicate the observed statistics, with a tendency of the calibrated parameters to inflate. These features might be due to the less efficient balance between the number of states and the largest shock exhibited by these exercises relative to the benchmark case. Concerning the EZW model, α holds close to its upper bound, whereas η oscillates remarkably across the different exercises. One may notice a slight improvement in $\sigma\{R_f\}$ and $E\{\Omega_m\}$ accompanied by a worsening of the remaining statistics of similar magnitude throughout the board.

Overall, the robustness exercises confirm the appropriateness of our benchmark calibration strategy. The marked quantitative sensitivity of the models' outcomes to variations of data targets and Markov chain parameterization is accompanied by the results remaining virtually intact from a qualitative perspective.

4 Final remarks and conclusion

We have investigated the effects of including strong time preference linkages into a macro-finance model. We have done so by analyzing the effects of incorporating an exogenous state variable on the representative consumer's choice regarding consumption and investment decisions. The state variable has introduced two elements in the stochastic discount factors of the fully specified approach: consumer confidence and year-on-year growth rates (on a quarterly data frequency). The year-on-year convention adopted to compute the growth rates may be interpreted as capturing potential habit formation; consumer confidence as the symmetric concept in an intertemporal perspective, in other words, as a way to capture potential utility from anticipation.

Our findings have indicated that the model compares favorably with the well-established contribution by Bansal and Yaron (2004) in terms of calibrated preference parameters (governing the subjective discount factor, relative risk aversion, and intertemporal elasticity of substitution) as well as concerning three (targeted) statistics, namely the mean and standard deviation of the risk-free rate, and the mean of the market return. In contrast, the model underperforms concerning three (untargeted) statistics, namely the standard deviation of the market return and the mean and volatility of the price-dividend ratio. Illustrative comparison of the overall performance of our approach relative to that of the Bansal-Yaron (BY) model may be produced by computing the average deviation of the six simulated moments from the relevant observed statistics, in absolute value and percentage terms. This calculation entails that the lower the score, the better the goodness of fit of the approach. The score obtained by the BY model is 18.96%, more than twice as small as the one calculated for our approach (38.83%).

We have considered three other model specifications to evaluate the impact of the two elements in isolation and the performance of the model that abstracts from both of them. We have found that disregarding either or both elements results in an acute deterioration of the model's performance. In relative terms, our results suggest that dropping consumer confidence is somewhat less detrimental than excluding habit persistence or discarding both elements (calculating the scores for these models yields 70.11%, 79.86%, and 85.73%, respectively). Finally, we have examined the effect of replacing the Consumer Confidence Index with the Consumer Sentiment Index to measure consumer confidence. Our results suggest that the models' performance using the two alternative measures is fairly comparable, with the latter recording a slightly higher score (41.38%).

The evidence we have produced indicates that our approach generally underperforms the more refined macro-finance literature frameworks. Nevertheless, the remarkable performance in replicating the observed equity premium and the first two unconditional moments of the risk-free rate indicates a potential role for the underlying time preference linkages in models incorporating long-run risks or rare or persistent-rare disasters. In particular, embedding time preference in those models might mitigate their exposure to the criticism raised by Epstein, Farhi and Strzalecki (2014) regarding the disproportionate "timing premium" (a measure of how much an individual would pay to have all risks resolved next period) implied by the existing versions of those models. We leave this matter for future research.

A Appendix

A.1 Derivation of the stochastic discount factor

Except for the preference specification (1), our framework is analogous to the Epstein-Zin-Weil (EZW) model: consumers' preferences are represented by a recursive utility function; two assets, one risk-free and the other state-contingent, are traded; free portfolio formation and the law of one price hold.

The representative consumer maximizes lifetime utility subject to the budget constraint

$$(p_{t+1} + y_{t+1})\, z_t + b_t \geq c_t + p_{t+1} z_{t+1} + q_{t+1} b_{t+1}$$

where b is the bond holding, q is the bond price, z is the stock holding, y is the stock dividend and p is the stock price. To ease notation, we denote the aggregate state with $s = (\kappa, \gamma, c, x)$. The variables involved in the determination of the state are levels and growth factors of the state variable and consumption, respectively related by the two equalities

$$\kappa_{t+1} = \gamma_{t+1} \kappa_t \text{ and } c_{t+1} = x_{t+1} c_t$$

with the pair (γ, x) following a Markov chain. Keeping this in mind, the representative consumer's dynamic program can be formalized as

$$v\{z_t, b_t, s_t\} = \max_{c_t, z_{t+1}, b_{t+1}} \left[(1-\beta)(\kappa_t c_t)^\rho + \beta \mu_{s_t} \{ v\{z_{t+1}, b_{t+1}, s_{t+1}\} \}^\rho \right]^{\frac{1}{\rho}}$$

subject to

$$(p\{s_t\} + y_t)\, z_t + b_t \geq c_t + p\{s_t\}\, z_{t+1} + q\{s_t\}\, b_{t+1}$$

where $\mu_s\{\cdot\}$ is the certainty equivalent conditional on the state s; likewise, the stock and bond prices, $p\{s\}$ and $q\{s\}$, are also conditional on the state s; $v\{\cdot\}$ is the representative consumer's value function conditional on the asset holdings z and b as well as on the state s.

Denote $W\{c, \mu\} = [(1-\beta)(\kappa c)^\rho + \beta \mu^\rho]^{\frac{1}{\rho}}$, and note that the partial derivatives are

$$\begin{aligned} W_c\{c, \mu\} &= \frac{1}{\rho} [(1-\beta)(\kappa c)^\rho + \beta \mu^\rho]^{\frac{1}{\rho}-1} (1-\beta) \rho \kappa^\rho c^{\rho-1} = (1-\beta)(W\{c,\mu\})^{1-\rho} \kappa^\rho c^{\rho-1} \\ W_\mu\{c, \mu\} &= \frac{1}{\rho} [(1-\beta)(\kappa c)^\rho + \beta \mu^\rho]^{\frac{1}{\rho}-1} \beta \rho \mu^{\rho-1} = \beta (W\{c,\mu\})^{1-\rho} \mu^{\rho-1} \end{aligned}$$

The partial derivative of μ_{s_t} with respect to z_{t+1} is

$$\frac{\partial}{\partial z_{t+1}}\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\} = \\ \left(\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}\right)^{\alpha} E_t\left\{[v\{z_{t+1},b_{t+1},s_{t+1}\}]^{-\alpha}\frac{\partial}{\partial z_{t+1}}v\{z_{t+1},b_{t+1},s_{t+1}\}\middle| s_t\right\}$$

The first-order condition (FOC) for the choice of z_{t+1} is

$$W_c\left\{c_t,\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}\right\}p\{s_t\} = \\ W_\mu\left\{c_t,\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}\right\}\frac{\partial}{\partial z_{t+1}}\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}$$

which we can write as

$$(1-\beta)(\kappa_t)^{\rho}(c_t)^{\rho-1}p\{s_t\} = \\ \beta\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}^{\rho-1+\alpha}E_t\left\{(v\{z_{t+1},b_{t+1},s_{t+1}\})^{-\alpha}\frac{\partial}{\partial z_{t+1}}v\{z_{t+1},b_{t+1},s_{t+1}\}\middle| s_t\right\}$$

We can use an envelope argument to get an expression for the derivative of v with respect to z. From the budget constraint we have

$$\frac{\partial}{\partial z}c\{z,\cdot\} = p\{s\}+y$$

At state (z_t,b_t,s_t), the derivative is given by

$$\begin{aligned}\frac{\partial}{\partial z_t}v\{z_t,b_t,s_t\} &= (1-\beta)\left(W\left\{c_t,\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}\right\}\right)^{1-\rho}(\kappa_t)^{\rho}(c_t)^{\rho-1}(p\{s_t\}+y_t)\\ &= (1-\beta)(v\{z_t,b_t,s_t\})^{1-\rho}(\kappa_t)^{\rho}(c_t)^{\rho-1}(p\{s_t\}+y_t)\end{aligned}$$

We now advance this expression one period and plug it into the right-hand side of the FOC to get the first-order condition for the holdings of the stock

$$(\kappa_t)^{\rho}(c_t)^{\rho-1}p\{s_t\} = \beta\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}^{\rho-1+\alpha}\cdot \\ E_t\left\{(v\{z_{t+1},b_{t+1},s_{t+1}\})^{1-\rho-\alpha}(\kappa_{t+1})^{\rho}(c_{t+1})^{\rho-1}(p\{s_{t+1}\}+y_{t+1})\middle| s_t\right\} \quad (14)$$

The first-order condition concerning the riskless asset is analogous and obtained simply by plugging in $q\{s_t\}$ for $p\{s_t\}$ and 1 for the payoff $p\{s_{t+1}\}+y_{t+1}$, obtaining

$$(\kappa_t)^{\rho}(c_t)^{\rho-1}q\{s_t\} = \\ \beta\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}^{\rho-1+\alpha}E_t\left\{(v\{z_{t+1},b_{t+1},s_{t+1}\})^{1-\rho-\alpha}(\kappa_{t+1})^{\rho}(c_{t+1})^{\rho-1}\middle| s_t\right\}$$

Imposing equilibrium (consumption must equal dividends, i.e., $c = y$; the representative household constantly holds all the stock, i.e. $z = 1$; but no bond, i.e., $b = 0$) and rearranging the model's asset pricing formulas for the equity price becomes

$$p\{s_t\} = E_t \left\{ \beta \left(\frac{V\{s_{t+1}\}}{\mu_{s_t}\{V\{s_{t+1}\}\}} \right)^{1-\rho-\alpha} \left(\gamma_{t+1}\right)^{\rho} \left(x_{t+1}\right)^{\rho-1} \left(p\{s_{t+1}\} + y_{t+1}\right) \middle| s_t \right\} \tag{15}$$

where to simplify notation we let $V\{s\} = v\{1, 0, s\}$, representing the representative consumer's value function in equilibrium. The right-hand side of equation (3) corresponds to the first four terms of the expectation operator's argument.

Iterative procedure

Since the pair (γ, x) is assumed to follow a Markov chain and lifetime utility is homogeneous of degree one in κc, the SDF depends only on (γ_t, x_t) and $\left(\gamma_{t+1}, x_{t+1}\right)$, with γ_t and x_t appearing just in the conditioning of the certainty equivalent. For some function ϕ, the equilibrium value function can be therefore written as

$$V\{s_t\} = \phi\{\gamma_t, x_t\} \kappa_t y_t \tag{16}$$

Plugging this expression into (2), we can rewrite the certainty equivalent operator as

$$\mu_t\{V\{s_{t+1}\}\} = \left(E_t \left\{ \left(\phi\left\{\gamma_{t+1}, x_{t+1}\right\} \gamma_{t+1} x_{t+1} \right)^{1-\alpha} \right\} \right)^{\frac{1}{1-\alpha}} \kappa_t y_t$$

The third term in (3) then becomes

$$\left(\frac{V\{s_{t+1}\}}{\mu_{s_t}\{V\{s_{t+1}\}\}} \right)^{1-\rho-\alpha} = \left(\frac{\phi\left\{\gamma_{t+1}, x_{t+1}\right\} \gamma_{t+1} x_{t+1}}{\mu_{\gamma_t, x_t}\left\{\phi\left\{\gamma_{t+1}, x_{t+1}\right\} \gamma_{t+1} x_{t+1}\right\}} \right)^{1-\rho-\alpha}$$

where $\mu_{\gamma_t, x_t}\left\{\phi\left\{\gamma_{t+1}, x_{t+1}\right\} \gamma_{t+1} x_{t+1}\right\} \equiv \left(E_t \left\{ \left(\phi\left\{\gamma_{t+1}, x_{t+1}\right\} \gamma_{t+1} x_{t+1} \right)^{1-\alpha} \right\} \right)^{\frac{1}{1-\alpha}}$, which in turn implies that the SDF reads

$$m\left\{\gamma_t, \gamma_{t+1}, x_t, x_{t+1}\right\} = \beta \left(x_{t+1}\right)^{\rho-1} \left(\gamma_{t+1}\right)^{\rho} \left(\frac{\phi\left\{\gamma_{t+1}, x_{t+1}\right\} \gamma_{t+1} x_{t+1}}{\mu_{\gamma_t, x_t}\left\{\phi\left\{\gamma_{t+1}, x_{t+1}\right\} \gamma_{t+1} x_{t+1}\right\}} \right)^{1-\rho-\alpha}$$

In this expression, x_{t+1} and ϕ are vectors, while m is the matrix

$$m\{j, k\} = \beta \left(x\{k\}\right)^{\rho-1} \left(\gamma\{k\}\right)^{\rho} \left(\frac{\phi\{k\} \gamma\{k\} x\{k\}}{\mu\{j\}} \right)^{1-\rho-\alpha} \tag{17}$$

where j denotes the current state, k the future state and

$$\mu\{j\} = \left(\sum_{k=1}^{S} \pi\{j,k\}\left(\phi\{k\}\gamma\{k\}x\{k\}\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha}} \tag{18}$$

with $\pi\{j,k\}$ indicating the transition probability from state j to state k.

Together with (15), (17) and (18) entail that the equity price is homogeneous in κc. Using (15), the price/dividend ratio, defined as $\omega\{s\} = p\{s\}/y$, can be written as

$$\omega\{j\} \equiv \frac{p\{j\}}{y\{j\}} = \sum_{k=1}^{S} \pi\{j,k\}\, m\{j,k\}\, x\{k\}\left(1+\omega\{k\}\right)$$

The bond price is simply

$$q\{j\} = \sum_{k=1}^{S} \pi\{j,k\}\, m\{j,k\}$$

We need to compute the matrix $m = m\{j,k\}$ to solve these equations. This task requires the calculation of the function ϕ. Here is where it becomes necessary to resort to the iterative procedure. Recall that $\phi\{\gamma, x\}\kappa y$ is the value function in equilibrium, which in turn represents the representative consumer's maximized lifetime utility. We can therefore write

$$\phi\{\gamma_t, x_t\}\kappa_t y_t = \left[(1-\beta)(\kappa_t y_t)^{\rho} + \beta\left(\mu_{\gamma_t,x_t}\left\{\phi\left\{\gamma_{t+1}, x_{t+1}\right\}\kappa_{t+1}y_{t+1}\right\}\right)^{\rho}\right]^{\frac{1}{\rho}}$$

which, dividing both sides by $\kappa_t y_t$, using $\kappa_{t+1} = \gamma_{t+1}\kappa_t$, $y_{t+1} = x_{t+1}y_t$ and the homogeneity of μ, becomes

$$\phi\{\gamma_t, x_t\} = \left[1-\beta+\beta\left(\mu_{\gamma_t,x_t}\left\{\phi\left\{\gamma_{t+1}, x_{t+1}\right\}\gamma_{t+1}x_{t+1}\right\}\right)^{\rho}\right]^{\frac{1}{\rho}}$$

For the generic state i, this expression corresponds to the vector

$$\phi\{i\} = \left[1-\beta+\beta\left(\mu\{i\}\right)^{\rho}\right]^{\frac{1}{\rho}} \tag{19}$$

with $\mu\{\cdot\}$ as in (18). To solve for ϕ, we treat the last two equations as a mapping that, at the k-th iteration, takes the vector ϕ^{k-1} into a new vector ϕ^k. Specifically: given ϕ^{k-1}, we first use (18) to generate a vector of certainty equivalents μ^k; we then use μ^k to obtain ϕ^k using (19). This two-step procedure is repeated iteratively until the change in ϕ produced by successive iterations is sufficiently small to be considered negligible.

A.2 Calibration of Markov chain and variables' realizations

In order to examine the quantitative aspects of asset pricing, we use a finite-state discrete Markov chain for the state variables. Specifically, we apply the method developed by Tauchen (1986) for choosing values for the state variables and the transition probabilities so that the resulting finite-state Markov chain mimics an underlying continuous-valued autoregression closely. The motivation for the method is the well-known fact that captures the statistical properties of the time series involved in the analysis adequately (after an adjustment for trend).

We begin by characterizing the vector autoregressive model. Let the growth rates of the M variables involved in the analysis be[24]

$$\begin{aligned} & g_t^v \equiv v_{t+1}/v_t, \text{ for } v = v_1, v_2, ..., v_M \text{ and } t = 1, 2, ..., T \\ & g_t \equiv \left[g_t^{v_1}, g_t^{v_2}, ..., g_t^{v_M}\right] \text{ for } t = 1, 2, ..., T \\ & g \equiv \left[g_1, g_2, ..., g_T\right]' \end{aligned} \tag{20}$$

where g_t is a $1 \times M$ vector collecting the growth rates at time t, and g a $T \times M$ matrix. Furthermore, let

$$\begin{aligned} & E\left(g^v\right) \equiv \tfrac{1}{T} \textstyle\sum_{t=1}^{T} g_t^v, \text{ for } v = v_1, v_2, ..., v_M \\ & E\left(g\right) \equiv \left[E\left(g^{v_1}\right), E\left(g^{v_2}\right), ..., E\left(g^{v_M}\right)\right] \end{aligned} \tag{21}$$

where $E\left(g\right)$ is a $1 \times M$ vector collecting the unconditional average growth rates. Finally, let

$$y_t \equiv g_t - E\left(g\right) \tag{22}$$

be generated by the vector autoregressive (VAR) process

$$y_t = A y_{t-1} + \epsilon_t \tag{23}$$

with the VAR error term covariance matrix represented by

$$var\left(\epsilon_t\right) = \Sigma_\epsilon \tag{24}$$

a diagonal $M \times M$ matrix, where A is the $M \times M$ matrix of VAR coefficients and ϵ_t is the $1 \times M$ vector of VAR white noise error terms at time t, with the v-th element denoted by ϵ_t^v. It is assumed that the elements of ϵ_t are mutually independent, each with distribution $\Pr\left[\epsilon_t^v \le u\right] = Z\left(u/\sigma\left(\epsilon^v\right)\right)$, where Z is the cumulative distribution of a standardized Gaussian process and $\sigma\left(\epsilon^v\right)$ is the standard deviation of the VAR error term ϵ^v.

We now turn to develop the structure of the finite-state discrete model. Let $\tilde{y}_t$ denote the approximating Markov chain vector for y_t in (23). Each component $\{\tilde{y}_t^v\}_{v=v_1,...,v_M}$ takes on one

[24] The number of variables equals the generic value M because our analysis requires the model to be solved for the consumption growth in isolation (one variable) as well as for the consumption growth rate in conjunction with the state variable growth rate (two variables).

of N values

$$\begin{aligned} &\bar{y}_1^v < \bar{y}_2^v < ... < \bar{y}_N^v, \text{ for } v = v_1, v_2, ..., v_M \\ &\bar{y}^v \equiv [\bar{y}_1^v, \bar{y}_2^v, ..., \bar{y}_N^v], \text{ for } v = v_1, v_2, ..., v_M \\ &\bar{y} \equiv [\bar{y}^{v_1}, \bar{y}^{v_2}, ..., \bar{y}^{v_M}]' \end{aligned} \tag{25}$$

where the generic value $\bar{y}_l^v$ is indexed by $l = 1, 2, ..., N$ and $\bar{y}$ is a $M \times N$ matrix.

A method for selecting the values of the components of $\bar{y}^v$ for each $v = v_1, v_2, ..., v_M$ is to let $\bar{y}_1^v$ and $\bar{y}_N^v$ be respectively minus and plus a small integer m times the unconditional standard deviation of y_t^v, with the remaining components satisfying

$$\bar{y}_{l+1}^v = \bar{y}_l^v + w^v, \text{ for } v = v_1, v_2, ..., v_M \text{ and } l = 1, 2, ..., N-1 \tag{26}$$

where

$$w^v = 2m\,\sigma(y^v) / (N-1), \text{ for } v = v_1, v_2, ..., v_M \tag{27}$$

The $\{\sigma(y^v)\}_{v=x,\gamma}$ are the square roots of the diagonal elements of the matrix Σ_y that satisfies $\Sigma_y = A\Sigma_y A' + \Sigma_\epsilon$, which can be found by iterating

$$\Sigma_y(r) = A\Sigma_y(r-1)A' + \Sigma_\epsilon \tag{28}$$

with convergence as $r \to \infty$ guaranteed so long as (23) is stationary.

There are N^M possible states for the system. Enumerate these states using the index $i = 1, 2, ..., N^M$. Let

$$\begin{aligned} &\bar{l}(i) \equiv \left[\bar{l}^{v_1}(i), \bar{l}^{v_2}(i), ..., \bar{l}^{v_M}(i)\right]', \text{ for } i = 1, 2, ..., N^M \\ &L \equiv \left[\bar{l}(1), \bar{l}(2), ..., \bar{l}\left(N^M\right)\right] \end{aligned} \tag{29}$$

where L is a $M \times N$ matrix, and $\bar{l}(i)$ is a $M \times 1$ vector of integers associated with state i such that, when the system is in state i at any given time t, the components of $\tilde{y}_t \equiv \tilde{y}(i)$ assume the values

$$\tilde{y}^v(i) = \bar{y}_{q^v}^v \text{ for } v = v_1, v_2, ..., v_M$$

where $q^v = \bar{l}^v(i)$. As a result, when the system is in state i at any given time t, $\tilde{y}_t = \bar{y}^i = \left[\bar{y}_{q^{v_1}}^v, \bar{y}_{q^{v_2}}^{v_2}, ..., \bar{y}_{q^{v_M}}^{v_M}\right]'$. We sort the states in such a way that, as the state index increases, the value of the component $\tilde{y}^{v_1}$ varies only after it has been matched with each value of the component $\tilde{y}^{v_2}$, which in turn varies only after it has been matched with each value of the component $\tilde{y}^{v_2}$, and so forth.

We wish to calculate the transition matrix $\pi(j,k) = \Pr\left[\tilde{y}_t = \bar{y}^k \,|\, \tilde{y}_{t-1} = \bar{y}^j\right]$. Let

$$\mu \equiv [\mu^{v_1}, \mu^{v_2}, ..., \mu^{v_M}]' = A\tilde{y}(j) \tag{30}$$

denote the impact of the lagged variables on of the realization of $\tilde{y}_t$ conditional on the state at time $t-1$ being j. For each v, let $h^v(j,l) = \Pr[\tilde{y}_t^v = \bar{y}_l^v \,|\, \text{state } j \text{ at } t-1]$, for $v = v_1, v_2, ..., v_M$, be the marginal probability that the v-th component of $\tilde{y}_t$ takes the value $\bar{y}_l^v$ conditional on observing the state j at time $t-1$; specifically, we define

$$\begin{aligned} h^v(j,l) &= Z(\bar{y}_l^v - \mu^v + w^v/2) - Z(\bar{y}_l^v - \mu^v - w^v/2) && \text{if } 2 \leq l \leq N-1 \\ &= Z(\bar{y}_1^v - \mu^v + w^v/2) && \text{if } l = 1 \\ &= 1 - Z(\bar{y}_N^v - \mu^v - w^v/2) && \text{if } l = N \end{aligned} \tag{31}$$

Given (31), the transition probabilities $\pi(j,k) = \Pr[\text{in state } k \,|\, \text{ in state } j]$ are, by independence of the ϵ^v, the products of the appropriate h^v,

$$\begin{aligned} \pi(j,k) &= \textstyle\prod_{v=x,\gamma} h^v\left(j, \bar{l}^v(k)\right), \text{ for } j,k = 1,2,...,N^M, \\ \pi &= \begin{bmatrix} \pi(1,1) & \pi(1,2) & ... & \pi\left(1,N^M\right) \\ \pi(2,1) & \pi(2,2) & ... & \pi\left(2,N^M\right) \\ ... & ... & ... & ... \\ \pi\left(N^M,1\right) & \pi\left(N^M,2\right) & ... & \pi\left(N^M,N^M\right) \end{bmatrix} \end{aligned} \tag{32}$$

where π is a $N^M \times N^M$ matrix.

The $1 \times N^M$ vector of unconditional probabilities is identified by any (e.g., the first) row of the matrix obtained by raising π in (32) to a power χ large at will,

$$p = I_{N^M} \cdot \pi^\chi \tag{33}$$

where $I_{N^M} \equiv [1, 0, ...]$ is a $1 \times N^M$ vector.

The realizations of the Markov chain corresponding to the future states characterizing the columns of π and p are identified using the matrix $\bar{y}$ in (25) in conjunction with the matrix L in (29). Specifically, for state i and variable v, the value $l^v(i)$ in L identifies the column of $\bar{y}$ storing, at the v-th row, the Markov chain realization relative to v in i. The value so identified must be added to the m-th column of the vector in (21) to obtain the i-th state realization of variable v's growth rate,

$$\begin{aligned} &\bar{g}^v(i) = \bar{y}^v(l^v(i)) + E(g^v), \text{ for } v = v_1, v_2, ..., v_M \text{ and } i = 1,2,...,N^M \\ &\bar{g}^v = \left[\bar{g}^v(1), \bar{g}^v(2), ..., \bar{g}^v\left(N^M\right)\right]', \text{ for } v = v_1, v_2, ..., v_M \\ &\bar{g} = [\bar{g}^{v_1}, \bar{g}^{v_2}, ..., \bar{g}^{v_{N^M}}] \end{aligned} \tag{34}$$

where $\bar{g}$ is a $N^M \times M$ matrix.

References

[1] Abel, A. B. (1990). Asset prices under habit formation and catching up with the Joneses. *American Economic Review, Papers and Proceedings*, 80, 38-42.

[2] Acemoglu, D. and Scott, A. (1994). Consumer confidence and rational expectations: Are agents' beliefs consistent with the theory? *Economic Journal*, 104, 1-19.

[3] Albuquerque, R., Eichenbaum, M., Luo, V. X. and Rebelo, S. (2016). Valuation risk and asset pricing. *Journal of Finance*, 71, 2861-2904.

[4] Andreasen, M. M. and Jørgensen, K. (2020). The importance of timing attitudes in consumption-based asset pricing models. *Journal of Monetary Economics*, 111, 95-117.

[5] Aschauer, D. A. (1985). Fiscal policy and aggregate demand. *American Economic Review*, 75, 117-127.

[6] Bansal, R., Kiku, D. and Yaron, A. (2012). An empirical evaluation of the long-run risks model for asset prices. *Critical Finance Review*, 1, 183-221.

[7] Bansal, R. and Yaron, A. (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance*, 59, 1481-1509.

[8] Barberis, N., Shleifer, A. and Vishny, R. (1998). A model of investor sentiment. *Journal of Financial Economics*, 49, 307-343.

[9] Barro, R. J. (2006). Rare disasters and asset markets in the twentieth century. *Quarterly Journal of Economics*, 121, 823-866.

[10] Barro, R. J. (2009). Rare disasters, asset prices, and welfare costs. *American Economic Review*, 99, 243-64.

[11] Barsky, R. B. and Sims, E. R. (2012). Information, animal spirits, and the meaning of innovations in consumer confidence. *American Economic Review*, 102, 1343-1377.

[12] Bathia, D. and Bredin, D. (2018). Investor sentiment: Does it augment the performance of asset pricing models? *International Review of Financial Analysis*, 59, 290-303.

[13] Bram, J. and Ludvigson, S. C. (1998). Does consumer confidence forecast household expenditure? A sentiment index horse race. *Economic Policy Review*, 4, 59-78.

[14] Campbell, J. Y. (2015). Emerging trends: Asset pricing. *Emerging Trends in the Social and Behavioral Sciences*, 1, 1-18.

[15] Campbell, J. Y. and Cochrane, J. H. (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of political Economy*, 107, 205-251.

[16] Campbell, J. Y., Lo, A. W. and MacKinlay, A. C. (1997). *The Econometrics of Financial Markets*. Princeton University Press.

[17] Caplin, A. and Leahy, J. (2001). Psychological expected utility theory and anticipatory feelings. *Quarterly Journal of Economics*, 116, 55-79.

[18] Carroll, C. D., Fuhrer, J. C. and Wilcox, D. W. (1994). Does consumer sentiment forecast household spending? If so, why? *American Economic Review*, 84, 1397-1408.

[19] Chung, S. L., Hung, C. H. and Yeh, C. Y. (2012). When does investor sentiment predict stock returns? *Journal of Empirical Finance*, 19, 217-240.

[20] Cochrane, J. H. (2005). *Asset Pricing*. Princeton University Press.

[21] Cochrane, J. H. (2017). Macro-finance. *Review of Finance*, 21, 945-985.

[22] Constantinides, G. M. (1990). Habit formation: A resolution of the equity premium puzzle. *Journal of political Economy*, 98, 519-543.

[23] Constantinides, G. M. (2017). Asset pricing: models and empirical evidence. *Journal of Political Economy*, 125, 1782-1790.

[24] Dumas, B., Kurshev, A. and Uppal, R. (2009). Equilibrium portfolio strategies in the presence of sentiment risk and excess volatility. *Journal of Finance*, 64, 579-629.

[25] Eichenbaum, M. S., Hansen, L. P. and Singleton, K. J. (1988). A time series analysis of representative agent models of consumption and leisure choice under uncertainty. *Quarterly Journal of Economics*, 103, 51-78.

[26] Epstein, L. G., Farhi, E. and Strzalecki, T. (2014). How much would you pay to resolve long-run risk? *American Economic Review*, 104, 2680-2697.

[27] Epstein, L. G. and Zin, S. E. (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, 57, 937-969.

[28] Frederick, S., Loewenstein, G. and O'donoghue, T. (2002). Time discounting and time preference: A critical review. *Journal of Economic Literature*, 40, 351-401.

[29] Hall, R. E. (1988). Intertemporal substitution in consumption. *Journal of Political Economy*, 96, 339-357.

[30] Hansen, L. P., Heaton, J. C. and Li, N. (2008). Consumption strikes back? Measuring long-run risk. *Journal of Political Economy*, 116, 260-302.

[31] Hansen, L. P. and Singleton, K. J. (1982). Generalized instrumental variables estimation of nonlinear rational expectations models. *Econometrica*, 50, 1269-1286.

[32] Ho, C. and Hung, C. H. (2009). Investor sentiment as conditioning information in asset pricing. *Journal of Banking and Finance*, 33, 892-903.

[33] Jagannathan, R. and Wang, Y. (2007). Lazy investors, discretionary consumption, and the cross-section of stock returns. *Journal of Finance*, 62, 1623-1661.

[34] Kreps, D. M. and Porteus, E. L. (1978). Temporal resolution of uncertainty and dynamic choice theory. *Econometrica*, 46, 185-200.

[35] Kuznitz, A., Kandel, S. and Fos, V. (2008). A portfolio choice model with utility from anticipation of future consumption and stock market mean reversion. *European Economic Review*, 52, 1338-1352.

[36] Lemmon, M. and Portniaguina, E. (2006). Consumer confidence and asset prices: Some empirical evidence. *Review of Financial Studies*, 19, 1499-1529.

[37] Loewenstein, G. (1987). Anticipation and the valuation of delayed consumption. *Economic Journal*, 97, 666-684.

[38] Lucas Jr, R. E. (1978). Asset prices in an exchange economy. *Econometrica*, 46, 1429-1445.

[39] Ludvigson, S. C. (2004). Consumer confidence and consumer spending. *Journal of Economic Perspectives*, 18, 29-50.

[40] Ludvigson, S. C. (2013). Advances in consumption-based asset pricing: Empirical tests. In *Handbook of the Economics of Finance*, 2, 799-906.

[41] Mehra, R. (2012). Consumption-based asset pricing models. *Annual Review of Financial Economics*, 4, 385-409.

[42] Mehra, R. and Prescott, E. C. (1985). The equity premium: A puzzle. *Journal of Monetary Economics*, 15, 145-161.

[43] Melino, A. and Yang, A. X. (2003). State-dependent preferences can explain the equity premium puzzle. *Review of Economic Dynamics*, 6, 806-830.

[44] Piazzesi, M., Schneider, M. and Tuzel, S. (2007). Housing, consumption and asset pricing. *Journal of Financial Economics*, 83, 531-569.

[45] Rietz, T. A. (1988). The equity risk premium a solution. *Journal of Monetary Economics*, 22, 117-131.

[46] Startz, R. (1989). The stochastic behavior of durable and nondurable consumption. *Review of Economics and Statistics*, 71, 356-363.

[47] Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters*, 20, 177-181.

[48] Thimme, J. (2017). Intertemporal substitution in consumption: A literature review. *Journal of Economic Surveys*, 31, 226-257.

[49] Wachter, J. A. (2013). Can time-varying risk of rare disasters explain aggregate stock market volatility? *Journal of Finance*, 68, 987-1035.

[50] Weil, P. (1989). The equity premium puzzle and the risk-free rate puzzle. *Journal of Monetary Economics*, 24, 401-421.