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# Skill-Biased Technical Change and Immiserizing Growth

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# Skill-Biased Technical Change and Immiserizing Growth*

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**Abstract**

The combination of consumer preferences, technological changes, and different income elasticities among goods and services can generate inequalities among agents leading to winners and losers. Inspired by these mechanisms, we pose the following research question: "Can immiserizing growth (IG) emerge within a skill-biased technical change (SBTC) framework where all agents display the same preferences, and they only differ in their skill level and consequently in the wage rate earned in the market?". The methodology adopted is constructing a general equilibrium model within an SBTC framework characterised by heterogeneous agents in skills endowed by identical Stone-Geary preferences. The non-homothetic feature of these preferences enables agents to consume the same bundles in different proportions as income increases, providing consumption patterns more coherent with the real world. Exploring the sensitivity of the model, I identify, as key results, the underlying drivers capable of triggering IG situations: the non-homotheticity, the elasticity of substitution between labour inputs, and differential SBTC across sectors. No IG arises calibrating the model to the U.S. industries and labour data from the EU KLEMS dataset. Performing a series of counterfactual experiments reveals that a decrease (increase) in the annual growth factor for the high-skilled (low-skilled) population leads to IG situations.

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# 1 Introduction

In classical economic growth literature, technological progress is associated with productivity improvements that benefit all workers and thus represents a driver for agents' income in the long run. However, recalling the concept of skill-biased technical change (SBTC), Violante (2008) suggests how shifts in production technology may favour skilled over unskilled workers. Ceteris paribus, SBTC induces a rise in the skill premium, that is, the ratio of skilled agents' wages to unskilled agents' wages. In these terms, it can be stated that technological progress may benefit more a subset of workers, implying distributional effects of economic growth.

One of SBTC's potential distributional effects is immiserizing growth (IG). Bhagwati (1958) explains that IG emerges when, in the presence of economic expansion, the growing country is unable to benefit from the increase in production and experiences an overall reduction in real income. In this case, the deterioration in terms of trade outweighs the positive effects of economic growth, affecting the country's overall well-being. Within this international trade context, Jones (1985) discusses that IG emerges when income effects dominate substitution effects, and the occurrence of distortions such as tariffs can lead to welfare paradoxes and income redistribution among countries. The considered phenomenon was generally studied in traditional two-country trade models, but in recent years its investigation moved from open (Matsuyama 2000, Li et al. 2010 and Collie 2012) to closed economies (Murphy 2016 and Chang et al. 2018) without focusing exclusively on one or the other. Murphy (2016), considering a closed economy framework, states that if economic growth is biased towards the rich agents' consumption bundle, the poor agents' welfare may fall. Consequently, while in an open economy, the decline in welfare affects the whole country, in closed economies, it affects only a part of the population or, in other words, a subset of agents. Therefore, the relevant literature does not seem to identify a single cause for the phenomenon univocally; indeed, it lists several underlying determinants and different ways in which it might occur.

In this paper, I study the emergence of IG theoretically and quantitatively in a closed economy framework. The main research question is: *"Can immiserizing growth emerge within an SBTC framework where all agents display the same preferences, and they only differ in their skill level, and consequently in the wage rate earned in the market?"*. An environment characterised by the presence of SBTC may lead to different welfare implications among the workers' population. Moreover, these heterogeneous outcomes for the agents may vary not only in magnitude but also in the sign of welfare change. In these conditions, in the shaping of IG phenomena, the SBTC and the specific model's features are crucial. For this purpose, I develop a general equilibrium model with two agent types (skilled and unskilled) featuring Stone-Geary preferences and two firms (IT-intensive and non-IT-intensive). On the supply-side there are two goods that are produced in two different sectors: a technologically advanced and a non-technologically advanced one. I build my model on Murphy (2016) by generalizing his setting. Murphy (2016) mainly employs an extreme version of non-homotheticity in which high- and low-skilled agents consume different bundles of goods and shows that in such a setting IG emerges when there is SBTC. This is mainly due to the fact that, by consuming only one good, the unskilled cannot benefit from SBTC, and their utility declines with technological change. In my model I assume that agents are characterised by Stone-Geary (SG) preferences in both types of goods, so they are identical in terms of utility. In these terms, the challenges and novelties of my research are to investigate whether IG emerges in a more general setting with respect to Murphy (2016) and to understand the factors and mechanisms underlying this phenomenon. I note that IG should still emerge here because unskilled workers consume a large share of the non-technologically advanced good, and so potentially they might not benefit from technological change, as in Murphy (2016).

The effects generated by the model depend on the magnitude of the different drivers, but the key mechanism can be summarized as follows. Improving production technology in the IT-intensive sector through SBTC increases the marginal productivity of high-skilled workers and their wages. Skilled agents use their increased wages to demand more goods and labour from the technologically advanced sector; this demand requires that some skilled workers leave the non-technologically advanced industry and enter to work in the technologically advanced one. If high-skilled and low-skilled workers are substitutable in both sectors, the non-IT-intensive sector can replace skilled workers who left the IT-intensive sector with some unskilled. But the negative impact caused by the loss of the skilled (more productive) is greater than the benefit of the unskilled (cheaper) who enter the non-technologically advanced sector. Under these circumstances, the non-IT-intensive sector experiences a decrease in output (the unskilled are less productive) and a decline in the welfare of the low-skilled (lower real wages and consumption). Note that the poor do not fully internalize the improved technology in the IT-intensive sector because non-homothetic preferences induce them to consume still a large share of the non-IT-intensive sector's goods.

Through proofs of concepts, I explore the sensitivity of the SG model and identify three main drivers that enable the model to display IG: the non-homotheticity, the elasticity of substitution between labour inputs, and SBTC's presence in one of the sectors. Increasing the non-homothetic parameter facilitates the presence of different consumption paths between skilled and unskilled and favours the emergence of IG phenomena.[1]An increase in the elasticity of substitution between labour inputs in the IT-intensive sector, with values greater than unity for both elasticities (the empirically relevant case), can lead to IG situations. Another case in which low-skilled agents experience a decrease in well-being is when the labour types in the non-IT-intensive sector become complements. However, it is relevant to maintain a certain degree of substitutability in either industry. In addition, SBTC is necessary: in the absence of higher technology growth in the IT-intensive sector, IG does not emerge.

I build an appropriate industry classification for my underlying theoretical framework and develop a quantitative section in which I calibrate the model with U.S. data from the EU KLEMS

[1]Indeed, Murphy (2016) is an extreme type of non-homotheticity in which one of the shares is degenerated at zero for one type of consumer, and the other share is degenerated to zero for the other type of consumer.

dataset. Simulating the calibrated model, no IG emerges. Both agents exhibit increasing utility over time. The calibrated non-homotheticity is small, and the model does not display the main drivers identified in the proofs of concepts capable of triggering IG.

I then run counterfactual experiments to study feasible scenarios from the calibrated model equilibrium. Working on two experiment lines, implemented to study the effects of a change in population (POP-line) and technology (TECH-line) growth factors, I find that different population compositions give rise to several IG cases. Specifically, a decrease (increase) in skilled agents (unskilled), relative to their calibrated values, leads to a reduction in the welfare of the low-skilled and, thus, to an IG case.

The remainder of the paper is as follows. Section 1.1 presents the model; Section 1.2 explores the model's sensitivity; Section 1.3 provides the quantitative analysis in which are shown the employed data, the datasets construction and the entire model calibration procedure; Section 1.4 runs counterfactual experiments. In Section 1.5, I conclude.

## 1.1 Model

This section presents a general equilibrium model in which two types of agents are characterized by identical non-homothetic preferences of the Stone-Geary type, and two firms competitively produce two different goods within a skill-biased technological change (SBTC) framework.

### 1.1.1 Setup

There are two types of agents: high-skilled ($H$) and low-skilled ($L$), defined by the type of labour they own. The two $J$-sector model is characterised by two firms, $Y$ and $F$, which competitively produce two goods $j = \{y, f\}$.[2] The $y$-good produced by $Y$ firm represents a bundle of technologically advanced goods (produced by IT-intensive industries); the $f$-good produced by $F$ firm represents a bundle of non-technologically advanced goods (produced by non-IT-intensive industries). For simplicity, I refer to the $y$-good as yachts and the $f$-good as potatoes. Technological change is exogenous. Time is discrete.

### 1.1.2 Agent's problem

Agents have identical non-homothetic preferences of the following form:

$$U_a = \left[\theta c_{f,a}^{\rho} + (1-\theta)\left(c_{y,a} + \bar{y}\right)^{\rho}\right]^{\frac{1}{\rho}},$$

[2] The general framework of the model is similar to Murphy (2016), but I differ from him because my representative agents have the same preferences and can consume the same bundle of goods. Murphy (2016) is an extreme version of non-homotheticity in which the two types of consumers consume entirely different bundles at their respective income levels. Even in the Cobb-Douglas case, agents' expenditure shares are different: the poor agent has an $\alpha$ expenditure share on the $f$-good; the rich agent has a $\beta$ expenditure share on the same $f$-good. Furthermore, my model does not adopt the rich and poor agent terminology, but I consider skilled and unskilled agents.

where: $a \in \{h = skilled,\, l = unskilled\}$ represents the agent's type; $c_f$ and $c_y$ are, respectively, the consumptions of an $(f)$ non-technologically advanced good (e.g., potatoes) and the consumptions of a $(y)$ technologically advanced good (e.g., yachts); $\theta \in \{0, 1\}$ is the weight of good $f$ in the utility function, the larger it is, the more the household will spend on $f$ good relative to $y$ good; $\bar{y} > 0$ is a non-homothetic preference parameter; $\rho$ is the parameter that governs the elasticity of substitution between the agent's goods $j$: if $\rho \to 1$ good $f$ and $y$ become perfect substitutes (indifference curves are straight lines); if $\rho \to -\infty$ they are perfect complements (indifference curves are L-shaped); if $\rho \to 0$ I obtain the Cobb-Douglas case. These Stone-Geary class preferences are non-homothetic because they permit different consumption proportions as income rises (an increasing Engel curve for the IT-intensive good and a non-increasing Engel curve for the other good).

The agent solves the following maximization problem:

$$\max_{\{c_{f,a}, c_{y,a}\}} U_a = \left[\theta c_{f,a}^{\rho} + (1-\theta)\left(c_{y,a} + \bar{y}\right)^{\rho}\right]^{\frac{1}{\rho}} \tag{P1}$$

subject to

$$P_f c_{f,a} + P_y c_{y,a} = w_a \ ,$$

where: $P_f$ and $P_y$ are the prices of goods $f$, $y$ and, $w_a$ is the total wage of the skilled/unskilled agent. Since the preferences are equal, the only thing that differentiates the two agents is their wage $w_a$.

### 1.1.3 Firm's problem

There are two sectors $J = \{Y, F\}$ in the economy, sector $Y$ contains the IT-intensive industries and produces a technologically advanced $y$-good; sector $F$ contains the non-IT-intensive industries and produces a non-technologically advanced $f$-good. In both sectors, goods are competitively produced. The two representative firms solve the following profit-maximization problems:

$$\max_{\{H_y, L_y\}} \Pi = \max_{\{H_y, L_y\}} \left[P_y Y - w_h H_y - w_l L_y\right] \tag{P2}$$

subject to

$$Y = \left[\mu\left(z_y H_y\right)^{\frac{\sigma_y - 1}{\sigma_y}} + (1-\mu)\, L_y^{\frac{\sigma_y - 1}{\sigma_y}}\right]^{\frac{\sigma_y}{\sigma_y - 1}} ;$$

$$\max_{\{H_f, L_f\}} \Pi = \max_{\{H_f, L_f\}} \left[P_f F - w_h H_f - w_l L_f\right] \tag{P3}$$

subject to

$$F = \left[ \eta \left( z_f H_f \right)^{\frac{\sigma_f - 1}{\sigma_f}} + (1 - \eta) L_f^{\frac{\sigma_f - 1}{\sigma_f}} \right]^{\frac{\sigma_f}{\sigma_f - 1}}$$

where: $\mu$ ($\eta$) represent the weight of high-skilled labour in the $Y$ ($F$) production function, and the larger it is, the more the high-skilled agent is used to produce $Y$ ($F$) goods; $z_j$ encloses the technology used by sector $J$ in producing good $j = \{y, f\}$; $H_j(L_j)$ is the high-skilled (low-skilled) labour employed in producing $j$; $\sigma_j$ is the elasticity of substitution between high-skilled and low-skilled labour in the $J$ sector and if $\sigma_j \to \infty$ the isoquant curves are inclined at 45°, if $\sigma_j \to 0$, the isoquant curves are right-angled (Leontief-style).

### 1.1.4 Equilibrium

A competitive equilibrium for the economy under study is a set of prices $\{P_{j=\{y,f\}}, w_{a=\{h,l\}}\}$, allocations for the agents $\{c_{j,a}\}$ and allocations for the firms $\{H_j, L_j\}$ such that, given prices: (i) $\{c_{j,a}\}$ solve the agent's maximization problem; (ii) $\{H_j, L_j\}$ solve the firm's maximization problems; (iii) all markets clear:

$$Y = Hc_{y,h} + Lc_{y,l},$$

$$F = Hc_{f,h} + Lc_{f,l},$$

$$L = L_f + L_y,$$

$$H = H_f + H_y.$$

## 1.2 Exploring the model's sensitivity

In this section, I explore the sensitivity of my theoretical model. The main purpose is understanding whether identical non-homothetic preferences can capture immiserizing growth phenomena within a skill-biased technological change context. Starting from analyzing a defined benchmark case, I develop a three-level proof of concept to investigate and highlight the underlying drivers of the IG phenomenon. The full model calibration will be explained and developed in section 1.3.

### 1.2.1 Baseline

I compare the Cobb-Douglas (CD) version of the Murphy (2016) model with my model characterized by Stone-Geary (SG) preferences. The values used for the considered benchmark case are reported in Table 1. Some parameters are model-specific: $\alpha$ and $\beta$ are characteristic for the Cobb-

| Parameter | Description | CD | SG |
|---|---|---|---|
| | *Model-specific parameters* | | |
| $\alpha$ | expend. share on good $f$ (unskilled) | 0.79 | - |
| $\beta$ | expend. share on good $f$ (skilled) | 0.61 | - |
| $\bar{y}$ | non-homothetic parameter | - | 0.20 |
| $\rho$ | elasticity of sub. for $c_f$ and $c_y$ | - | 0.05 |
| $\theta$ | weight of good $f$ in the utility | - | 0.50 |
| | *Elasticities and weights* | | |
| $\sigma_y$ | elasticity of sub. ($H$, $L$) in $Y$ | 2 | 2 |
| $\sigma_f$ | elasticity of sub. ($H$, $L$) in $F$ | 2 | 2 |
| $\mu$ | weight of $H_y$ (skilled agents) in $Y$ | 0.50 | 0.50 |
| $\eta$ | weight of $H_f$ (skilled agents) in $F$ | 0.50 | 0.50 |
| | *Stock and time* | | |
| $z_y$, $z_f$ | technology stock in 1970 | 1 | 1 |
| $H/L$ | ratio of $H/L$ population in 1970 | 0.15 | 0.15 |
| $t$ | time (from 1970 to 2015) | 45 | 45 |
| | *Growth factors* | | |
| $gZ_Y$ | annual growth factor of tech. in $Y$ | 1.05 | 1.05 |
| $gZ_F$ | annual growth factor of tech. in $F$ | 1.00 | 1.00 |
| $\gamma_h$ | annual growth factor of $HS$ pop. | 1.00 | 1.00 |
| $\gamma_l$ | annual growth factor of $LS$ pop. | 1.00 | 1.00 |

Table 1: CD and SG models in the benchmark case.

Douglas specification, while $\theta$ and $\bar{y}$ are specific for the Stone-Geary specification.[3] I assume the presence of SBTC in only one sector by setting the annual growth factor of technology in sector $Y$ greater than the growth factor in sector $F$: $gZ_Y > gZ_F$. Both sectors have the same initial technology stock ($z_y = z_f = 1$). The weight of good $f$, in the Stone-Geary utility function, is $\theta = 0.5$, and the elasticity of substitution between $H$ and $L$ workers is the same in the two sectors ($\sigma_y = \sigma_f = 2$). This elasticity choice is empirically relevant in the literature (Katz and Murphy (1992), Angrist (1995), Krusell et al. (2000)). The unskilled population in the initial year (1970) is normalised to 1, so the skill ratio is equal to $\frac{H}{L} = 0.15$ (data re-elaborated from the EU KLEMS dataset) and can be read more easily. At this theoretical study stage, I keep the initial population composition constant in the economic system. Thus the annual growth factors of the skilled and unskilled population $\gamma_{a=\{h,l\}}$ are set to one.[4]

Figures 1-2 show the utility functions of the skilled and unskilled agents in the two models (CD and SG) using the parameter values explained and given in Table 1. Values used in the symmetric case (Figure 1) are essentially the same as those used in the benchmark case (Figure

[3]In the Cobb-Douglas version of the model in Murphy (2016), even if agents can consume both goods, they have different expenditure shares. The poor agent has an $\alpha$ expenditure share on the good potato; the rich agent, conversely, has a $\beta$ expenditure share on the same good potato.

[4]In the quantitative analysis of section 1.3, all parameters will be calibrated, and the annual population growth factors will not be constant but directly observed from the real data.

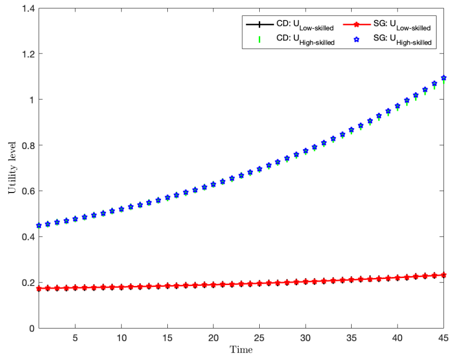


Figure 1: Symmetric case.
*Notes*: if $\bar{y} = 0 \Rightarrow U_a^{CD} = U_a^{SG}$.

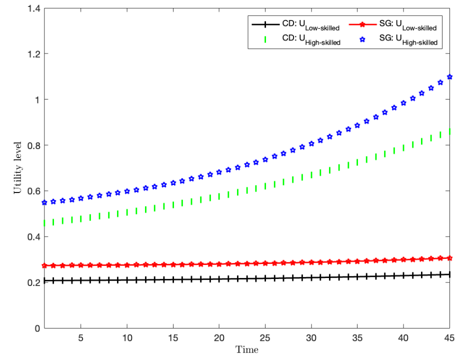


Figure 2: Benchmark case.
*Notes*: if $\bar{y} > 0 \Rightarrow U_a^{CD} \neq U_a^{SG}$.

2). The only difference lies in the value of the non-homothetic parameter (equal to zero in the symmetric case and equal to 0.2 in the benchmark case) and, consequently, in the initial value of the expenditure shares of the CD model ($\alpha = \beta = 0.50$ in Figure 1 and $\alpha = 0.79$ and $\beta = 0.61$ in Figure 2). This choice has been adopted to show, graphically and as a first approximation, the influence of non-homotheticity. In the presence of a 5 per cent annual growth of technology in the $Y$ sector, in the symmetric case, the utility functions have the same trend in the two models: between the period $t = 1970$ and the period $T = 2015$, the high-skilled and low-skilled agents utility increase (on average between the CD and SG configurations) by 121 per cent and 23 per cent, respectively. The SBTC in sector $Y$ increases the relative productivity of the skilled relative to the unskilled, increasing the skill premium. This technological advance benefits both agents, even with different magnitudes, because now the models are equal (non-homotheticity has been set to zero). In the benchmark case, on the other hand, the non-homothetic parameter is set greater than zero ($\bar{y} = 0.2$). Although agents (skilled/unskilled) have the same preferences (i.e., they can consume both types of goods), they now consume these goods in different proportions depending on their income level. Figure 2 shows how the presence of non-homotheticity in the SG preferences allows capturing in a better way (compared to CD preferences) the positive effect of SBTC in the $Y$-sector. The utility function of skilled (blue line) and unskilled (red line) agents in the SG model, has larger values than the utility functions (green and black line) in the CD model.

I explain, as follows, the general procedure used for the simulations contained in the entire section 1.2. Starting from the SG benchmark scenario in Table 1, I modify a single parameter in the SG model to see the new initial values in the consumption shares of good $f$ for skilled and unskilled agents. I then put these initial share values into the $\alpha$ and $\beta$ parameters of the CD model, and I modify the same parameter in the CD model (if it is not a model-specific parameter). In other words, I impose the same initial agent consumption level (equal consumption shares in the first period) and simulate the models simultaneously; this allows us to study the variations in the agents' utility functions of the two models as the same parameter changes.[5] To be more precise, the parameters in common between the two models must be the same; only one parameter changes (in both) at a time and the initial expenditure shares are updated and set at the same initial level in both models.

### 1.2.2 Proof of concept (a): role of non-homotheticity

The use of preferences with a non-homothetic component allows agents, contrary to homothetic preferences, to consume goods in different proportions, even if tastes are the same. This feature translates into expenditure shares that are not constant along the income path; in other words, the expenditure becomes an income function. As income increases, the consumption of the available goods in the economic system will increase following different consumed proportions. Under this premise, I want to understand whether non-homotheticity can also play a role within the IG phenomenon and, therefore, whether it can be identified as one of its underlying drivers. Therefore, I perform the first line (a) of the proof of concept by considering the effect of a change in the non-homothetic parameter $\bar{y}$ on the agents' utility functions. Since IG involves a decrease in utility (relative to a supposed starting situation), my benchmark remains the one with $\bar{y} = 0.20$ (and all parameter values are given in Table 1). From this baseline: (i) I derive the initial ($t = 1970$) and final ($T = 2015$) values for the utility functions of skilled ($h$) and unskilled ($l$) agents in the two different configurations (Cobb-Douglas and Stone-Geary); (ii) I set $\bar{y} = 0$ to study the effect of removing the non-homotheticity; (iii) I set $\bar{y} = 0.6$ to study the effect of increasing the non-homotheticity. When running these counterfactuals, the only parameters that vary are $\alpha$ and $\beta$ in the Murphy model because they represent the expenditure shares (for skilled and unskilled agents, respectively) in that model and I align them to the initial shares in the non-homothetic model. In period $t = 1970$, I put them equal to those in the SG configuration so that both models have an identical starting expenditure shares scenario between the agents.

[5] I set the same initial consumption shares of good $f$ for the skilled and unskilled agents in both models. The parameters $\alpha$ and $\beta$ in the CD model incorporate this information and enable automatic calculation of the shares of the other good $y$ as $1 - \alpha$ (for the unskilled agent) and $1 - \beta$ (for the skilled agent).

| | Benchmark | | Proof of concept (a) | | | |
|---|---|---|---|---|---|---|
| | | | $\bar{y}=0$ | | $\bar{y}=0.6$ | |
| **Variables** | **t=1970** | **T=2015** | t | T | t | T |
| $U_h^{CD}$ | 0.4588 | 0.8602 | 0.4477 | 1.0821 | 0.5769 | 0.7176 |
| $U_l^{CD}$ | 0.2074 | 0.2344 | 0.1734 | 0.2319 | 0.3468 | 0.3415 |
| $U_h^{SG}$ | 0.5477 | 1.0976 | 0.4477 | 1.0949 | 0.7477 | 1.1149 |
| $U_l^{SG}$ | 0.2734 | 0.3064 | 0.1734 | 0.2318 | 0.4570 | 0.4486 |
| **Parameters** | | | | | | |
| $\alpha$ | 0.79 | | 0.50 | | 1 | |
| $\beta$ | 0.61 | | 0.50 | | 0.84 | |

Table 2: The role of non-homotheticity.

*Notes*: $\alpha$ and $\beta$ are the model-specific parameters in the Cobb-Douglas configuration. $\alpha$ represents the expenditure share of good $f$ for the low-skilled agent; $\beta$ represents the expenditure share of good $f$ for the high-skilled agent.

Table 2 summarizes the results obtained. In the case of removing non-homotheticity ($\bar{y}=0$), the initial values (column $t$) in the utility functions of all agents have lower values than in the benchmark. This is reasonable since the expenditure shares have decreased ($\alpha$ from 0.79 to 0.50 and $\beta$ from 0.61 to 0.50), and the agents' utility functions capture the lower amount of goods purchased. However, between $t$ and $T$, utilities increase, so there are no unexpected effects.

Instead, in the case of increasing the non-homotheticity ($\bar{y}=0.6$), I observe an increase (between 1970 and 2015) in the skill agents' utility functions and a decrease in the unskilled agents' utility functions; this is an IG phenomenon. In particular, the unskilled's utility decreases by 1.53% from $U^{CD}_{l,t=1970}=0.3468$ to $U^{CD}_{l,T=2015}=0.3415$; while in the SG model, the unskilled's utility decreases by 1.84% (from $U^{SG}_{l,t=1970}=0.4570$ to $U^{SG}_{l,T=2015}=0.4486$). Now, a bunch of agents (unskilled) have lower utility levels than the initial situation despite, in one of the sectors, there is the presence of SBTC. Note how the increase in non-homotheticity changes the initial value of the expenditure share on good $f$; in fact, now $\alpha=1$ brings us back to the economic mechanism found in Murphy (2016). He explains that when the poor agent consumes only the $f$ bundle goods ($\alpha=1$) produced by sector $F$, and when the technology is biased toward sector $Y$, the welfare of poor agents decreases. This happens because their price index increases due to the increased price of skilled labour.

What is novel in my work is that I can replicate this result using identical non-homothetic preferences among agents without imposing different expenditure shares a priori. Specifically, in the SG model, I only increased non-homotheticity. This increment in non-homotheticity facilitates different consumption paths. Even having identical preferences, now, the unskilled agents consume only one type of good (the $f$ non-technologically advanced one), facing a decline in their welfare, represented by their utility function. Furthermore, the presence of non-homotheticity seems to increase the magnitude of the immiserizing growth phenomenon: the percentage decrease in the unskilled's utility in the SG model is higher than in the CD model, and this is shown by the equations below, calculating the percentage change in the unskilled utility, in both models, between 1970 and 2015.

$$\Delta\% U_l^{SG} = \left( \frac{U_{l,T=2015}^{SG} - U_{l,t=1970}^{SG}}{U_{l,t=1970}^{SG}} \right) \cdot 100 = \qquad \left( \frac{0.4486 - 0.4570}{0.4570} \right) \cdot 100 = -\,1.84\,,$$

$$\Delta\% U_l^{CD} = \left( \frac{U_{l,T=2015}^{CD} - U_{l,t=1970}^{CD}}{U_{l,t=1970}^{CD}} \right) \cdot 100 = \qquad \left( \frac{0.3415 - 0.3468}{0.3468} \right) \cdot 100 = -\,1.53\,,$$

$$\left(\Delta\% U_l^{SG} = -1.84\right) > \left(\Delta\% U_l^{CD} = -1.53\right).$$

### 1.2.3 Proof of concept (b): role of the elasticity of substitution

In both models, $\sigma_y$ ($\sigma_f$) represents the elasticity of substitution between labour $H$ and $L$ in sector $Y$ ($F$). According to the literature, the value of these two elasticities in the benchmark is set to $\sigma_y = \sigma_f = 2$ so that the empirically relevant case is when the labour type (skilled and unskilled) are substitutes and thus have values greater than unity. Knowing from Murphy (2016) the necessity to maintain at least $\sigma_y > 1$ to generate a decrease in the welfare of the poor (in my model, unskilled agents), I want to understand whether, in general, $\sigma_y$ and $\sigma_f$ can lead to any IG cases. Starting from the benchmark utility values, we perform two types of exercises. In the proof of concept (b.1) of Table 3, (i) we set $\sigma_y$ below unity ($\sigma_y = 0.5$), and then (ii) we set $\sigma_y$ above unity ($\sigma_y = 8$). In the proof of concept (b.2) of Table 4, I repeat the same exercises changing $\sigma_f$ this time. The reason is to study the effects of elasticity of substitution on the agents' utility levels; the methodology employed to carry out the simulations is the same as previously explained.

Looking at the values in Table 3, I note that reducing $\sigma_y$ below unity, as mentioned in Murphy (2016), does not allow us to find cases of IG; the values of the utilities of low-skilled agents ($U_l$), in $T = 2015$, are not lower than the values in $t = 1970$. The situation changes when I set $\sigma_y$ to a higher value than the benchmark case; with $\sigma_y = 8$, an IG case emerges in both models. The value of $U_{l,T=2015}^{CD} = 0.2157$ in the final year is lower than the initial value ($U_{l,t=1970}^{CD} = 0.2181$), revealing a disutility. The same trend for $U_l^{SG}$, which decreases from a value of 0.2969 to 0.2868. Specifically, the model with Cobb-Douglas preferences records a decline in utility for low-skilled agents of 1.10%. In comparison, the model with Stone-Geary preferences records a drop in utility for low-skilled agents of 3.40%. Again, as in the proof of concept (a), the Stone-Geary configuration, compared to the Cobb-Douglas configuration, seems to capture a larger portion of the decrease in the unskilled's well-being: $\left([\Delta\%]_{T-t}\, U_l^{SG} = 3.40\right) > \left([\Delta\%]_{T-t}\, U_l^{CD} = 1.10\right)$. The only circumstance in which this is not the case is illustrated by Table 4. In this proof of concept (b.2), I deviate from the empirically relevant case to test the presence of complementarity between production inputs in sector $F$. I obtain an IG case in both models with $\sigma_f$ below unity ($\sigma_f = 0.5$). Low-skilled agents' utility reduces by 20.57% in the $CD$ model (from $U^{CD}_{l,t=1970} = 0.1011$ to $U^{CD}_{l,T=2015} = 0.0803$) and by 6.9% in the $SG$ model (from $U^{SG}_{l,t=1970} = 0.1710$ to $U^{SG}_{l,T=2015} = 0.1592$). This difference occurs since the lack of non-homotheticity of the CD configuration does not allow the emergence of different consumption paths. Thus, low-skilled agents experience a greater decrease in their welfare. In contrast, no decline emerges in the unskilled's well-being when I pose substitutability with $\sigma_f = 8$

The key economic mechanism of the model can be explained as follows: within a framework characterized by the presence of SBTCs in the IT-intensive sector $Y$, the marginal productivity of $H$ working in this sector is higher than that of $L$, and consequently, their wages are also higher. $H$ agents demand greater quantities of both $H$ labour and $y$-good, increasing their demand and causing skilled agents to move out of sector $F$ to work in sector $Y$. With $\sigma_f > 1$, in sector $F$, it is possible to replace $H$ workers (who have left the sector) with $L$ workers who work in $Y$. Still, in this case, the effect of the outflow of skilled labour from sector $F$ is more significant than the inflow of unskilled labour. Thus, the result is a decrease in output in $F$ and a decline in the welfare of low-skilled agents. On the other hand, with $\sigma_f < 1$, $H$ and $L$ agents are complements, and the outflow of skilled labour from $F$ lowers the value of $L$, and their wages also decrease relative to the wages of the rich, leading to a decrease in welfare for the low-skilled.

This economic mechanism consistently underlies the two IG cases observed in Tables 3 and 4; with sufficiently high elasticity of substitution ($\sigma_y = 8$), even keeping $\sigma_f$ at the benchmark values ($\sigma_f = 2$), the outflow of $H$s from $F$ is higher than the inflow of $L$s. The final effect is a decrease in the welfare of the unskilled (Table 3). On the other hand, keeping $\sigma_y = 2$ as in the benchmark case and making the types of workers complements in sector $F$, the outflow of $H$s from $F$ lowers the value of $L$s and results in a decrease in their welfare (table 4). In this way, I find that the elasticity of substitution between labour types is another driver of IG.

| | | | **Proof of concept (b.1)** | | | |
|---|---|---|---|---|---|---|
| | **Benchmark** | | $\sigma_y = 0.5$ | | $\sigma_y = 8$ | |
| **Variables** | **t=1970** | **T=2015** | t | T | t | T |
| $U_h^{CD}$ | 0.4588 | 0.8602 | 0.4890 | 0.6590 | 0.4167 | 1.1194 |
| $U_l^{CD}$ | 0.2074 | 0.2344 | 0.2275 | 0.2670 | 0.2181 | 0.2157 |
| $U_h^{SG}$ | 0.5477 | 1.0976 | 0.5717 | 0.8192 | 0.5054 | 1.3787 |
| $U_l^{SG}$ | 0.2734 | 0.3064 | 0.2592 | 0.3258 | 0.2969 | 0.2868 |
| **Parameters** | | | | | | |
| $\alpha$ | 0.79 | | 0.89 | | 0.72 | |
| $\beta$ | 0.61 | | 0.63 | | 0.61 | |

Table 3: The role of the elasticity of substitution ($\sigma_y$).

*Notes*: in the benchmark, $\sigma_y$ is equal to 2.

| Variables | Benchmark t=1970 | T=2015 | Proof of concept (b.2) $\sigma f = 0.5$ t | T | $\sigma f = 8$ t | T |
|---|---|---|---|---|---|---|
| $U_h^{CD}$ | 0.4588 | 0.8602 | 0.7481 | 1.1876 | 0.3344 | 0.6816 |
| $U_l^{CD}$ | 0.2074 | 0.2344 | 0.1011 | 0.0803 | 0.2539 | 0.3378 |
| $U_h^{SG}$ | 0.5477 | 1.0976 | 0.8372 | 1.2757 | 0.4157 | 1.0782 |
| $U_l^{SG}$ | 0.2734 | 0.3064 | 0.1710 | 0.1592 | 0.3249 | 0.3980 |
| **Parameters** | | | | | | |
| $\alpha$ | 0.79 | | 0.84 | | 0.74 | |
| $\beta$ | 0.61 | | 0.54 | | 0.67 | |

Table 4: The role of the elasticity of substitution ($\sigma_f$).

*Notes*: in the benchmark, $\sigma_f$ is equal to 2.

### 1.2.4 Proof of concept (c): role of SBTC

Violante (2008) describes the SBTC as a technological variation favouring skilled over unskilled labour. Murphy (2016) states that if economic growth is biased toward the rich's consumption bundle, the well-being of the poor may decline. The framework in which I found the three IG cases (the previous proof of concepts) is characterised by the presence of SBTC in only one of the sectors. Specifically, I used as a default situation the one where $(gZ_Y = 1.05) > (gZ_F = 1)$. That is because $Y$ is a sector that contains IT-intensive industries, and it is reasonable for it to have a higher technology growth factor than sector $F$, which includes non-IT-intensive industries.

In this section, I test, in two different ways, the necessity of SBTC to elicit IG phenomena. (i) I set the annual technology growth factors at the same level in both sectors ($gZ_Y = gZ_F = 1.01$); (ii) I set the yearly technology growth factor of sector $F$ at a higher level than that of sector $Y$: $(gZ_Y = 1) < (gZ_F = 1.05)$. The results are reported in Table 5. The first column shows as interest variables the low-skilled agents' utility functions in the two models; the second and third columns, bracketed under the heading "Immiserizing growth," show the initial (in $t = 1970$) and final (in $T = 2015$) utility function values.[6] Note how these are the IG cases found during the previous exercises. From the table, I can note that, following the application of exercises (i) and (ii), the agents' utility functions increase by eliminating the three reported cases of IG.

These results provide an understanding of the importance of specific SBTC in $Y$. Such a productive improvement in the IT-intensive sector can activate the model's economic mechanism. Hence, high-skilled agents, having a better marginal productivity performance, have higher wages, which they use to purchase more quantities of skilled labour and technologically advanced goods. Depending on the elasticity of substitution between labour types ($\sigma_j$) in the production functions

[6] The agent utility functions under investigation are represented by monotonic functions. For this reason and the simplicity of exposition, I dwell on the initial/final values. This choice in no way changes the performed analysis.

and the level of the non-homotheticity ($\bar{y}$) in the utility functions, the effects that deteriorate the well-being of low-skilled agents can arise, as explained in the previous section. In contrast, when this initial feature is not present, the specific chain of economic mechanisms is not activated, and IG does not appear. I agree with Murphy (2016) about the necessity of sector-specific technological improvement and identify SBTC as a critical driver in the formation of IG.

| | **Immiserizing growth** | | **Proof of concept (c)** | |
|---|---|---|---|---|
| | | | $gZ_Y = gZ_F$ | $gZ_Y < gZ_F$ |
| **Variables** | **t=1970** | **T=2015** | T | T |
| *Proof of concept (a):* $y = 0.6$ | | | | |
| $U_l^{CD}$ | 0.3468 | 0.3415 | 0.5332 | 0.5538 |
| $U_l^{SG}$ | 0.4570 | 0.4486 | 0.5657 | 0.5569 |
| *Proof of concept (b.1):* $\sigma_y = 8$ | | | | |
| $U_l^{CD}$ | 0.2181 | 0.2157 | 0.2854 | 0.3360 |
| $U_l^{SG}$ | 0.2969 | 0.2868 | 0.3575 | 0.4048 |
| *Proof of concept (b.2):* $\sigma_f = 0.5$ | | | | |
| $U_l^{CD}$ | 0.1011 | 0.0803 | 0.3659 | 0.4178 |
| $U_l^{SG}$ | 0.1710 | 0.1592 | 0.3801 | 0.3966 |

Table 5: The role of SBTC.

*Notes*: in the benchmark, $gZ_Y = 1.05$ and $gZ_F = 1$. In the column $gZ_Y = gZ_F$: both annual growth factors are set at 1.01. In the column $gZ_Y < gZ_F$: the value of $gZ_Y$ is one, while that of $gZ_F$ is 1.05.

### 1.2.5 Proof-of-concept highlights

In the previous subsections, I identified three main drivers capable of leading to IG scenarios. In proof of concept (a), I showed how a certain amount of non-homotheticity facilitates the presence of different consumption paths between agents characterised by identical preferences. Indeed, if non-homotheticity is reduced or removed, the IG also vanishes. In proof of concept (b), I introduced the importance of the elasticity of substitution between labour types ($H$, $L$). Initially respecting the empirically relevant case (values of both elasticities greater than unity), I showed how an increase in $\sigma_y$ favours the formation of IG. Subsequently, another IG situation is generated by keeping only $\sigma_y$ above unity and rendering complements in the $F$-sector, the agent's $H$ and $L$. In proof of concept (c), I demonstrate, on the other hand, how the presence of SBTC in the most technologically advanced sector is critical to operating in an appropriate framework for generating IG. In fact, without this feature, ceteris paribus, it is no longer possible to achieve IG situations.

I can claim that IG is a problematic phenomenon to identify, caused by a simultaneous combination of factors. Thus, it is necessary not to approach this problem by focusing only on some feature but rather to apply a new approach that I can call holistic. Concretely, the application of this new concept is expressed in the combined use of all previously identified drivers; ideally, the sum of the different drivers can provide a more significant result than the use of a single element. The aim is to find/create a framework capable of identifying different IG phenomena as broadly as possible.

A quantitative analysis is performed in the next section to bring the model to the data and increase its consistency. The construction of the datasets coherent with the setup outlined in Section 1.1, the target identification, and the model calibration will permit advancement in the detail of the analysis.

## 1.3 Quantitative analysis

This section describes the calibration strategy adopted to compare the model with the data. We explain the datasets used, the new industries classification employed, the calibrated parameters and their respective moments. The intention is to endow my theoretical model with more consistency, approximating the reality satisfactorily and analyzing the simulations resulting from using the new calibrated parameter values.

### 1.3.1 Data

I introduce the set of U.S. data that I use to calibrate the model. The first dataset is from EU KLEMS, Jäger (2017), where the variables are divided into values, prices, volumes, and additional variables. The detail level consists of 34 industries plus 8 aggregates released following the industry classification (NACE Rev. 2/ISIC Rev. 4). Appendix F. EU KLEMS provides complete detail of the industries and aggregates in the data.

I apply a new industries classification to customize the dataset for my research. I proceed as described below: by looking at all the industries in the dataset, I identify those in which the use of computers, software and communications equipment is prevalent. These industries are Information and communication; Financial and insurance activities; Professional, scientific, technical, administrative and support service activities; Transport and storage; Postal and courier activities. I then create the data counterpart of the model sector $Y$, which contains these IT-intensive industries and calculate the macro sector total value-added (VA) as the sum of the VA of the respective industries that compose it. The remaining industries in the dataset where IT technology is not prevalent (i.e. those not included in sector $Y$) are clustered within a new macro-sector $F$, which contains all non-IT-intensive industries. The sector $F$'s total value added (as in the case of sector $Y$) is calculated as the sum of the VA of its component industries, and these industries are as follows: Agriculture, forestry and fishing; Mining and quarrying; Food products, beverages and tobacco; Textiles, wearing apparel, leather and related products; Wood and paper products; printing and reproduction of recorded media; Coke and refined petroleum products; Chemicals and chemical products; Rubber and plastics products, and other non-metallic mineral products; Basic metals and fabricated metal products, except machinery and equipment; Electrical and optical equipment;

Machinery and equipment n.e.c.; Transport equipment; Other manufacturing; repair and installation of machinery and equipment; Electricity, gas and water supply; Construction; Wholesale and retail trade and repair of motor vehicles and motorcycles; Wholesale trade, except of motor vehicles and motorcycles; Retail trade, except of motor vehicles and motorcycles; Accomodation and food service activities; Real estate activities; Public administration and defence; compulsory social security; Education; Health and social work; Arts, entertainment and recreation; Other service activities.

The second source of data I use still comes from EU KLEMS and is the labour input data (United States-SIC based), release date March 2008.[7] This dataset contains, in detail, the following variables: shares in total hours worked in percentages by sex (male, female), age (15-29, 30-49, 50 and over) and by skill (high-skill, medium-skill and low-skill); total hours worked by employees (in millions).[8,9]

I also apply some adjustments to this dataset to make it consistent with my theoretical model. I only allow agents of two types: $H$ (high-skilled: college graduate and above) and $L$ (low-skilled: high school and some years of college + less than high school and some years of high school). Since the dataset provides shares in total hours worked (in percentages) per agents' category, their sum in year $t$ and sector $s$ $(H_t^s + L_t^s)$ is 100. Then, by multiplying the shares in total hours worked $(H_t^s,\ L_t^s)$ by the total hours worked by employees in millions $(H\_EMPE_t^s)$, I obtain a measure of the total hours worked by agents per sector and year. This quantity permits us to aggregate and, at the same time, split the economy into two aggregates: $H\_H_{t,j}^s$ (annual labour hours in sector $J = \{Y, F\}$ by high-skilled) and $H\_L_{t,j}^s$ (annual labour hours in sector $J = \{Y, F\}$ by low-skilled).

Having classified both datasets into two macro sectors (one with IT-intensive industries and the other with non-IT-intensive industries), I calculate the following indices and statistics for the entire period 1970-2015: value-added (VA), VA volume, VA price indices; Törnqvist of real quantities; shares and hours worked by $H$ and $L$ agents.[10] The Törnqvist index, which I will use to identify two moments in the calibration process, measures the variation of quantities between time $t$ and

[7] I use the 2008 version because it contains the shares in total hours worked by sex, age and skill (necessary characteristics for my research). The more recent 2021 version of Bontadini et al. (2021) (regarding the shares) contains only those related to employment by type in total industry employment and those related to compensation by type in total industry labour compensation.

[8] Definition of the people's skills in the EU KLEMS dataset. High-skilled: college graduate and above; medium-skilled: high school and some years of college (but not completed); low-skilled: less then high school and some years of high school (but not completed).

[9] Additional details on the contents and construction of the EU KLEMS dataset are provided in O'Mahony and Timmer (2009).

[10] Since the labour input dataset contained data only up to 2005, I extended the dataset to 2015 by using the trend growth until 2005. More specifically, I calculated the average growth rate and the growth factor, sector by sector, for the period 2001-2005 (the years after 2000 in which the use of technology increases) and assumed that the years 2006-2015 grow at that sector-specific growth factor. No extension was made in the other dataset, as the years I needed were already present.

time $t-1$ and is calculated as follows:

$$\frac{Q_t}{Q_{t-1}} = \prod_{s=1}^{S} \left( \frac{q_{s,t}}{q_{s,t-1}} \right)^{\frac{1}{2}\left[ \frac{p_{s,t-1}q_{s,t-1}}{\Sigma_{k=1}^{S}\left(p_{k,t-1}q_{k,t-1}\right)} + \frac{p_{s,t}q_{s,t}}{\Sigma_{k=1}^{S}\left(p_{k,t}q_{k,t}\right)} \right]},$$

where: $q_{s,t}$ represents the VA quantity of sector $s$ at time $t$; $p_{s,t}$ represents the VA price of sector $s$ at time $t$; $s$ and $k$ are subscripts that both indicate each industry (from 1 to $S = 29$) within the macro sectors $(Y, F)$; $t$ represents a single year given the whole time horizon 1970-2015. From the previous equation, it is straightforward to switch to the logarithmic form:

$$log\left(\frac{Q_t}{Q_{t-1}}\right) = log\left\{ \prod_{s=1}^{S} \left( \frac{q_{s,t}}{q_{s,t-1}} \right)^{\frac{1}{2}\left[ \frac{p_{s,t-1}q_{s,t-1}}{\Sigma_{k=1}^{S}\left(p_{k,t-1}q_{k,t-1}\right)} + \frac{p_{s,t}q_{s,t}}{\Sigma_{k=1}^{S}\left(p_{k,t}q_{k,t}\right)} \right]} \right\},$$

$$log\left(\frac{Q_t}{Q_{t-1}}\right) = \sum_{s=1}^{S} \left\{ \frac{1}{2} \left[ \frac{p_{s,t-1}q_{s,t-1}}{\sum_{k=1}^{S}\left(p_{k,t-1}q_{k,t-1}\right)} + \frac{p_{s,t}q_{s,t}}{\sum_{k=1}^{S}\left(p_{k,t}q_{k,t}\right)} \right] log\left(\frac{q_{s,t}}{q_{s,t-1}}\right) \right\},$$

$$log\left(\frac{Q_t}{Q_{t-1}}\right) = \frac{1}{2} \sum_{s=1}^{S} \left[ \frac{p_{s,t-1}q_{s,t-1}}{\sum_{k=1}^{S}\left(p_{k,t-1}q_{k,t-1}\right)} + \frac{p_{s,t}q_{s,t}}{\sum_{k=1}^{S}\left(p_{k,t}q_{k,t}\right)} \right] log\left(q_{s,t} - q_{s,t-1}\right).$$

### 1.3.2 Calibration

**Discussion** In order to simulate the model, it is necessary to calibrate nine parameters. Technology, in my model, is exogenous: given the initial stock ($z_y$, $z_f$), I have to figure out at what rate it grows year by year. Therefore I need to calibrate the total growth rates of technology in sector $Y$ ($gZ_Y$) and sector $F$ ($gZ_F$). For $gZ_Y$, I identify the US total relative wage growth rate (skilled/unskilled) between 1970-2015, observed in the EU KLEMS data, as the first target. This choice is reasonable since an increase in technology in sector $Y$ leads to an increase in the marginal productivity of skilled agents and, consequently, a rise in their wages. In my model, the US total relative wage growth rate is calculated as

$$M(1) = \left( \frac{w_{h,t=2015}}{w_{l,t=2015}} / \frac{w_{h,t=1970}}{w_{l,t=1970}} \right) - 1,$$

where: $M\left(\#\right)$ is the number of moments in my model; $\frac{w_{h,t=2015}}{w_{l,t=2015}}$ and $\frac{w_{h,t=1970}}{w_{l,t=1970}}$ are, respectively, the relative wages of skilled/unskilled agents in the final (2015) and the initial (1970) year.

For $gZ_F$, I identify as second target the total growth rate for the relative price $P_f/P_y$. In my model this moment is calculated as

$$M(2) = \left( \frac{P_{f,t=2015}}{P_{y,t=2015}} / \frac{P_{f,t=1970}}{P_{y,t=1970}} \right) - 1,$$

where: $P_f$ and $P_y$ are the prices of goods $f,\ y$. Again, the choice of this target lies in the broad role of technology: indeed, as $gZ_F$ changes, it is reasonable to assume that there are also effects on the prices of labour employed and goods produced in sector $F$.

The parameter $\rho$ governs the elasticity of substitution between the agent's goods ($c_y,\ c_f$); therefore, I identify the growth rate of $y$ quantities (represented by the Törnqvist index in $Y$) as the third target. I consider it an opportune moment because it considers the change between 2015 and 1970 in the quantities of all industries within macro sector $Y$, and these production decisions embody increments or decrements of market demand. In the model, this moment is calculated as

$$M(3) = \frac{Y_{2015}}{Y_{1970}} - 1.$$

Non-homotheticity is controlled by the $\bar{y}$ parameter; its role in the context of the non-homothetic identical preferences is to permit the expenditure share relative to the bundle of IT-intensive goods to grow with income. Thus, I expect variations in the quantities of the $f$ bundle (relative to the $y$ bundle). For this reason, my fourth moment is the growth rate of $f$ quantities represented by the Törnqvist index in $F$, which considers the change between 2015 and 1970 in the volume of all subsectors $F$. This target is calculated as follows

$$M(4) = \frac{F_{2015}}{F_{1970}} - 1.$$

Since $\theta$ is a fixed weight within the utility function, I identify the relative expenditure of goods $y/f$ in the initial year 1970 as the fifth moment. In addition, given that in my model, everything produced is consumed (there is no investment), then the target within the model is constructed as the nominal share of production Y in the initial year 1970 divided by the nominal share of production $F$ in the initial year 1970:

$$M(5) = \frac{nom\_share\_production\_y_{1970}}{nom\_share\_production\_f_{1970}}.$$

The parameter $\mu$ ($\eta$) represents the weight of high-skilled labour in sector $Y$ ($F$), while $1 - \mu$ ($1 - \eta$) represents the weight of low-skilled labour in sector $Y$ ($F$). From the FOCs of my model, it is possible to derive two equations in which the ratio of the above parameters is a function of the ratio of the labour type employed.

$$\frac{\mu}{\eta} = f\left(\frac{H_{y,t}}{H_{f,t}}\right); \qquad \frac{1-\mu}{1-\eta} = f\left(\frac{L_{y,t}}{L_{f,t}}\right).$$

Since $H_j$ and $L_j$ affect both the parameters $\mu$ and $\eta$ (which represent weights), then to calibrate $\mu$ ($\eta$), I identify as the sixth (seventh) target the ratio of the $H$ ($L$) labour quantity in sector $Y$ to the $H$ ($L$) labour quantity in sector $F$, both in the base year 1970.

$$M(6) = \frac{H_{y,t=1970}}{H_{f,t=1970}}; \qquad M(7) = \frac{L_{y,t=1970}}{L_{f,t=1970}}.$$

As for $\sigma_j$, it determines the degree of substitutability between $H$ and $L$ labour in sector $J$; I identify as the eighth (ninth) moment the difference between the amount of $H$ ($L$) labour in sector $Y$ with respect to the amount of $H$ ($L$) labour in sector $F$ both in the final year (2015). The reason is that comparing the sixth and seventh moments with the eighth and ninth moments enables us to observe the variations between the labour quantities during 1970-2015.

$$M(8) = \frac{H_{y,t=2015}}{H_{f,t=2015}}; \qquad M(9) = \frac{L_{y,t=2015}}{L_{f,t=2015}}.$$

**Results** Table 6 shows the calibrated parameters and targets illustrated previously. In detail: the first column, "Parameter", shows the list of my model's parameters; the second column, "Value", contains the values of the parameters resulting from the calibration process; the third column, "Target", displays a brief textual description of the moments identified; the fourth column "Model" illustrates the value of the moments generated by the model; the fifth column, "Data", contains the value of the moments identified directly from the data. In summary, looking at the last two columns (Model and Data), it is possible to see that the model fits the data reasonably well. The perfectly replicated moments by the model are the value of the total growth rate for the US relative wage, the total growth rate for the relative price and the ratio of $H$ workers in sector $Y$ to $H$ workers in sector $F$ in 1970. Almost perfectly replicated are the targets relating to the ratio of $L$ workers in sector $Y$ to $L$ workers in sector $F$ in 1970 and the difference between labour quantity by skill in both sectors, considering the final (2015) year. I believe the model replicates the remaining targets sufficiently well, except for the fifth moment, representing the level of relative expenditure in the year 1970.

I use the calibrated parameters to simulate my model characterized by Stone-Geary preferences over the period considered in the dataset construction. A complete information set depicts this simulation. I have all US data from the EU KLEMS dataset, I know the amount of $H$ and $L$

| Parameter | Value | Target | Model | Data |
|---|---|---|---|---|
| $gZ_Y$ | 0.54 | total relative wage growth rate | 0.23 | 0.23 |
| $gZ_F$ | 0.35 | total growth rate for $P_f/P_y$ | 0.14 | 0.14 |
| $\rho$ | 0.03 | growth of quantities in $Y$ (Törnqvist) | 3.68 | 5.05 |
| $\bar{y}$ | 0.02 | growth of quantities in $F$ (Törnqvist) | 2.19 | 1.97 |
| $\theta$ | 0.70 | relative expend. in $Y$ and $F$ (1970) | 0.09 | 0.21 |
| $\mu$ | 0.62 | ($H_y$ in 1970) / ($H_f$ in 1970) | 0.08 | 0.08 |
| $\eta$ | 0.63 | ($L_y$ in 1970) / ($L_f$ in 1970) | 0.09 | 0.10 |
| $\sigma_y$ | 3.07 | ($H_y$ in 2015) / ($H_f$ in 2015) | 0.11 | 0.09 |
| $\sigma_f$ | 4.38 | ($L_y$ in 2015) / ($L_f$ in 2015) | 0.11 | 0.10 |

Table 6: Calibrated parameters and targets.

*Notes*: $gZ_Y = 0.54$ and $gZ_F = 0.35$ represent the total growth rates over the period 1970-2015. Their corresponding annual growth factors are: $gZ_Y = 1.010$ and $gZ_F = 1.007$.

population in the initial and final year (1970-2015), and I also know their annual growth factors ($\gamma_h$ for the $H$ population and $\gamma_l$ for the $L$ population). The low-skilled population in 1970 is normalized to 1, so the $H/L$ ratio can be read more easily. The graphical representation of the utility functions by agent type (skilled and unskilled) is shown in Figure 3. The specific values used are shown in Table 7.

| Parameter | Description | Value |
|---|---|---|
| | *Elasticities and weights* | |
| $\sigma_y$ | elasticity of sub. ($H$, $L$) in $Y$ | 3.07 |
| $\sigma_f$ | elasticity of sub. ($H$, $L$) in $F$ | 4.38 |
| $\rho$ | elasticity of sub. for $c_f$ and $c_y$ | 0.03 |
| $\theta$ | weight of good $f$ in the utility | 0.70 |
| $\mu$ | weight of $H_y$ (skilled agents) in $Y$ | 0.62 |
| $\eta$ | weight of $H_f$ (skilled agents) in $F$ | 0.63 |
| | *Stock and non-homotheticity* | |
| $z_y$, $z_f$ | technology stock in 1970 | 1 |
| $H/L$ | ratio of $H/L$ population in 1970 | 0.15 |
| $\bar{y}$ | non-homothetic parameter | 0.02 |
| | *Time and growth factors* | |
| $t$ | time (from 1970 to 2015) | 45 |
| $gZ_Y$ | annual growth factor of tech. in $Y$ | 1.010 |
| $gZ_F$ | annual growth factor of tech. in $F$ | 1.007 |
| $\gamma_h$ | annual growth factor of $H$ pop. | 1.041 |
| $\gamma_l$ | annual growth factor of $L$ pop. | 1.007 |

Table 7: Stone Geary model using calibrated parameters.

I notice how, using the parameter values resulting from the calibration, both agents' utility functions (skilled and unskilled) are monotonically increasing over the entire period considered.

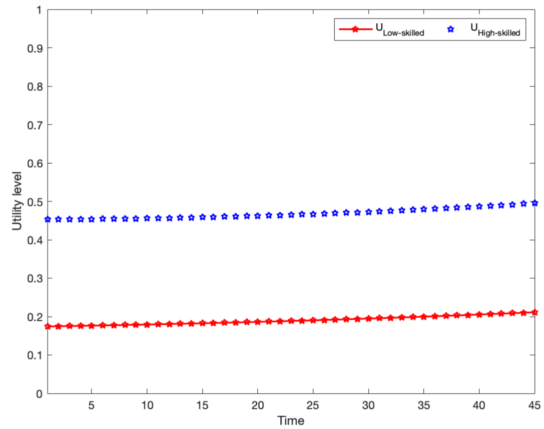


Figure 3: SG benchmark post-calibration process.

*Notes*: all parameter values are listed in table 7.

These trends are consistent with a slight annual improvement in technological progress in both sectors ($gZ_Y = 1.01$ and $gZ_F = 1.007$). In addition, a minimum non-homotheticity value of $\bar{y} = 0.02$ to the value of $c_{y,h} = 0.22$ in the first period evidences a situation characterized by little ability to generate different consumption patterns among agents. Thus, IG does not emerge as a result of the calibration process. The calibrated parameter values do not adequately leverage the drivers of the phenomenon. Now, SBTC is present in both the IT-intensive and non-IT-intensive sectors, and thus the outflow of $H$ workers from the $F$-sector to enter the $Y$-sector does not have the same previous quantitative magnitude. At the same time, the $Y$-sector is not the only one with high substitutability among production inputs. Instead, $H$ and $L$ in the $F$-sector are more easily replaceable (allowing for easy filling of the gap created by high-skilled, who may have left the $F$-sector). The low presence of non-homotheticity does not drive different consumption proportions. From this quantitative section, I understand how important the starting conditions of an economic system are to whether or not immiserizing growth phenomena are created.

In the next section, I perform counterfactual exercises to extend the analysis more broadly, considering the calibrated model as the starting point of the investigation.

## 1.4 Counterfactual analysis

As illustrated in section 1.2, the main drivers identified as capable of capturing IG phenomena are the non-homotheticity in preferences, the elasticity of substitution $\sigma_j$, and a framework characterized by the presence of SBTC in one of the sectors. With the calibration of the model, I found that a case of IG does not emerge quantitatively. As explained in the previous section, the calibrated parameter values do not enable us to draw a suitable framework for the phenomenon under investigation.

In this section, taking the results of the calibrated model as a baseline, I develop two lines of counterfactual exercises to deepen the analysis. The first line, POP-line, studies the effects of changes in the population composition; in other words, examining variations in the annual growth factors of the high-skilled and the low-skilled explores their impact on the variables of interest. The second line, the TECH-line, studies the effects of variation in the annual growth factors of technology in the two sectors. In the following, the details are given.

### 1.4.1 POP-line experiment

The POP-strand is characterized by a 2x3 framework in which I study the two annual population growth factors in 3 different situations. I assume that during the period of the simulations 1970-2015: (I) the annual population growth factor is constant $\Gamma^{cc}_{a=\{h,l\}} = \gamma_a = 1$, where $\Gamma^{cc}_{a=\{h,l\}}$ is the annual population growth factor value incorporating the scenario assumed by the counterfactual exercise; (II) the annual population growth factor declines by 5% ($\Gamma^{cc}_{a} = \gamma_a - 5\%$); (III) the annual population growth factor increases by 5% ($\Gamma^{cc}_{a} = \gamma_a + 5\%$). The superscript $cc$ (counterfactual) indicates that the parameter has assumed a new value relative to the benchmark; this value is given by the sum of the benchmark value, and the value of the hypothesized change occurs during the specific counterfactual exercise.

$$\Gamma^{cc}_{h} = \begin{cases} \gamma_h = 1 & \Rightarrow \ \textit{no growth in H population} \\ \gamma_h - 5\% & \Rightarrow \ \textit{H decline} \ \ (\textit{with } \Gamma^{cc}_{h} < 1) \\ \gamma_h + 5\% & \Rightarrow \ \textit{H growth} \ \ (\textit{with } \Gamma^{cc}_{h} > 1) \end{cases} ;$$

$$\Gamma^{cc}_{l} = \begin{cases} \gamma_l = 1 & \Rightarrow \ \textit{no growth in L population} \\ \gamma_l - 5\% & \Rightarrow \ \textit{L decline} \ \ (\textit{with } \Gamma^{cc}_{l} < 1) \\ \gamma_l + 5\% & \Rightarrow \ \textit{L growth} \ \ (\textit{with } \Gamma^{cc}_{l} > 1) \end{cases} .$$

Each scenario is generated by starting from the calibrated model (Figure 3 and Table 7) and changing only one parameter at a time.

The first two counterfactual exercises (I, II) generate similar trends in the variations of the variables. Closing the skilled population growth channel ($\Gamma^{cc}_{h} = \gamma_h = 1$) or applying a 5% decrease to the same growth factor ($\Gamma^{cc}_{h} = \gamma_h - 5\% = 0.99$) results in both cases in lower values for $\gamma_h$ than the benchmark. Exercise II produces an IG case. Table 8 shows the initial ($t = 1970$) and final ($T = 2015$) values for the utility functions, consumption, relative wages and production functions, following the various counterfactual exercises. The column $\gamma_h - 5\%$ shows what happens to the variables of interest with exercise II. Ceteris paribus, a decrease in $H$ growth of 5% causes a reduction in the quantities produced in both sectors ($Y^{benchmark}_{T=2015} = 0.37$ to $Y^{cc}_{2015} = 0.13$; $F^{benchmark}_{T=2015} = 0.91$ to

$F^{cc}_{2015} = 0.38$). With fewer $H$ (the most productive agents), less is produced in both sectors. The relative wages of $H$ increase (these agents are growing more slowly than $L$, and thus their marginal productivity increases); the relative wages of $L$ decrease (they are the less productive agents, and this is reflected in their lower wages). The consumption of the $L$ also decreases, and their utility function reflects all this information. In essence, I are dealing with an IG case: the high-skilled agents face an increase in relative wages, consumption and utility function; the low-skilled agents, on the other hand, experience a decrease in relative wages, consumption and finally, in the utility function, finding themselves in a worse situation overall than at the start.[11]

The third counterfactual exercise (III) considers the case in which the population growth factor increases by 5%. Specifically, I analyse the situation where the unskilled increase faster, represented by the column $\gamma_l + 5\%$. What happens to my variables of interest can be summarised as follows. With $\Gamma^{cc}_l = \gamma_l + 5\% = 1.06$, low-skilled agents increase relative to the benchmark level. An increase in $L$ reduces the marginal productivity of the factor, and as a consequence, the wages of unskilled agents also drop ($w_l/P_j$). With lower wages, $L$ consume less of both goods ($c_{y,l}$, $c_{f,l}$) and their utility function decreases ($U_l$). Thus, I are facing another IG in which, due to technological progress in both sectors ($gZ_Y = 1.010$; $gZ_F = 1.007$), only one category of agents can benefit from it: the high-skilled are more productive and have higher relative wages, they demand more significant quantities of goods by increasing the amounts produced in both sectors and consumption, and their utility is growing over time; on the other hand, low-skilled agents experience a decrease in relative wages that is reflected in lower consumption for both types of goods and finally in their lower utility function compared to the initial period.[12]

From this set of counterfactual exercises, I observe that even starting from a situation where both agents have increasing utility functions (benchmarks with calibrated values), a decrease in $H$s or an increase in $L$s can generate two IG cases. What I learn is that the population composition ($H/L$) and their growth factors ($\gamma_h$, $\gamma_l$) play a critical role in the possibility of generating IG. For this reason, they must be included among the drivers of the phenomenon.

### 1.4.2 TECH-line experiment

A 2x3 framework characterizes the TECH-strand; the aim is to study the two technology growth factors under three different scenarios. I assume that: (I) the annual technology growth factor is constant $GZ^{cc}_{J=\{Y,F\}} = gZ_J = 1$, where $gZ_J$ is the value from Table 7 and $GZ^{cc}_{J=\{Y,F\}}$ is the annual technology growth factor accounting for the specific counterfactual experiment; (II) the annual

[11] The utility function provides us with a depiction of the satisfaction level of a representative agent. I observe that $L$ agents, as a result of this counterfactual exercise, have lower utility than the benchmark. If in the benchmark, the final utility value was $U_{l,T=2015} = 0.2113$, now as a result of the $\gamma_h$ decrease, it is $U^{cc}_{l,T=2015} = 0.1706$. This value is also lower than the starting level ($U_{l,t=1970} = 0.1743$), and configures an IG situation.

[12] As a result of the increase in $\gamma_l$, the unskilled utility vector has a value of $U^{cc}_{l,T=2015} = 0.1702$ in the last period. It is also smaller than the value in the initial period of the benchmark case $U_{l,t=1970} = 0.1743$, shaping another IG case.

technology growth factor decreases by 5% compared to the benchmark ($GZ_J^{cc} = gZ_J - 5\%$); (III) the annual technology growth factor increases by 5% over the benchmark ($GZ_J^{cc} = gZ_J + 5\%$).

$$GZ_Y^{cc} = \begin{cases} gZ_Y = 1 & \Rightarrow \ \textit{no growth in technology stock} \\ gZ_Y - 5\% & \Rightarrow \ z_y \ \textit{decline} \ \ (\textit{with } GZ_Y^{cc} < 1) \\ gZ_Y + 5\% & \Rightarrow \ z_y \ \textit{growth} \ \ (\textit{with } GZ_Y^{cc} > 1) \end{cases} ;$$

$$GZ_F^{cc} = \begin{cases} gZ_F = 1 & \Rightarrow \ \textit{no growth in technology stock} \\ gZ_F - 5\% & \Rightarrow \ z_f \ \textit{decline} \ \ (\textit{with } GZ_F^{cc} < 1) \\ gZ_F + 5\% & \Rightarrow \ z_f \ \textit{growth} \ \ (\textit{with } GZ_F^{cc} > 1) \end{cases} .$$

I adopt the same technique as before: starting from the benchmark (Table 7), I change the value of only one parameter at a time.

The first counterfactual exercise (I), with $GZ_J = gZ_J = 1$, implies that the stock of available technology in sector $J = Y, F$ remains constant over time. The growth absence leads to similar results in both sectors. In the case of $GZ_Y = 1$ (in the IT-intensive industries), the output of $Y$ decreases as it experiences no technological innovation; on the other hand, the production in sector $F$ increases as there is a certain amount of technological innovation in the benchmark ($gZ_F = 1.007$). Conversely, with $GZ_F = 1$ (in the non-IT-intensive sector), the output of $Y$ increases due to the presence of a certain amount of technological advancement in the benchmark ($gZ_Y = 1.010$) and the output of $F$ decreases. However, closing the technology channel in any $J$ sector leads to decreased utilities for both agents (skilled and unskilled). Looking at the utility vectors in the final period, when closing the technology channel in sector $Y$ ($GZ_Y = 1$), they change from $U_{l,T=2015} = 0.2123$ of the benchmark to $U_{l,T=2015}^{cc} = 0.2066$ of the counterfactual and from $U_{h,T=2015} = 0.49$ of the benchmark to $U_{h,T=2015}^{cc} = 0.45$. Looking instead at these same vectors, when closing the technology channel in sector $F$ ($GZ_F = 1$), they shift from $U_{l,T=2015} = 0.2113$ in the benchmark to $U_{l,T=2015}^{cc} = 0.2089$ in the counterfactual and from $U_{h,T=2015} = 0.49$ in the benchmark to $U_{h,T=2015}^{cc} = 0.41$. In both cases, I observe a decrease in utilities for both skilled and unskilled agents.

The same trends are also confirmed by the second counterfactual exercise (II), in which $GZ_J = gZ_J - 5\%$. Bringing below unity (0.96), the values of the annual growth factors of technology (-5% to the benchmark) create a situation of sector-specific technological decline, which reasonably leads to a decrease in both agent utility functions.

The opposite results arise when, in the third counterfactual exercise (III), a 5% higher growth than in the benchmark case is assumed; in this case, all agents benefit from the technological improvement present in one of the sectors, showing an increasing utility function to the benchmark. In this particular situation, technology growth in sector $Y$ increases output in $Y$ but leads to lower levels in $F$ (from $F_{T=2015} = 0.91$ to $F^{cc}_{T=2015} = 0.88$). Coming back to the mechanism explained above, the presence of SBTC in the $Y$ IT-intensive sector increases the marginal productivity of skilled agents and their wages. These agents use their higher wages to demand more significant quantities of goods produced by sector $Y$, increasing demand for them; this induces some skilled workers in $F$-sector to leave that sector and work in $Y$-sector. Depending on the magnitude of this phenomenon and the elasticity of substitution between the production inputs in sector $F$, one may or may not experience a decrease in the $F$-sector's production.

In the TECH-line experiment, differently from the POP-line, I do not observe any IG cases, but this does not mean that technology is not a driver of IG. When there is the presence of SBTC in the IT-intensive sector and the presence of any of the other drivers discovered in section 1.2, it is easier to find some IG situations. On the other hand, as seen above in proof of concept (c), if the technology growth factors are not the same in both sectors, or if growth is faster in the non-IT-intensive sector, then IG does not occur. In this section, I do not observe this phenomenon because the growth factors of both sectors ($gZ_Y = 1.010$ and $gZ_F = 1.007$) are very similar, so there is no SBTC in only the $Y$ sector. Then, the presence of SBTC in the most technologically advanced sector is a crucial assumption to trigger the entire model mechanism and reveal IG situations.

| | | | Counterfactual exercises | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Benchmark | | $\gamma_h$ | | | $\gamma_l$ | | | $gZ_Y$ | | | $gZ_F$ | | |
| Var | 1970 | 2015 | =1 | -5% | +5% | =1 | -5% | +5% | =1 | -5% | +5% | =1 | -5% | +5% |
| | | | *Variable values in T=2015* | | | | | | | | | | | |
| $U_h$ | 0.45 | 0.49 | 0.63 | 0.69 | 0.44 | 0.48 | 0.44 | 0.67 | 0.45 | 0.40 | 0.92 | 0.41 | 0.16 | 2.32 |
| $U_l$ | 0.17 | 0.21 | 0.18 | 0.17 | 0.31 | 0.22 | 0.32 | 0.17 | 0.21 | 0.21 | 0.33 | 0.21 | 0.23 | 0.43 |
| $c_{y,h}$ | 0.22 | 0.26 | 0.33 | 0.36 | 0.23 | 0.26 | 0.23 | 0.35 | 0.20 | 0.13 | 2.04 | 0.25 | 0.22 | 0.43 |
| $c_{y,l}$ | 0.07 | 0.10 | 0.07 | 0.07 | 0.16 | 0.11 | 0.16 | 0.07 | 0.08 | 0.06 | 0.73 | 0.12 | 0.32 | 0.06 |
| $c_{f,h}$ | 0.59 | 0.63 | 0.81 | 0.89 | 0.56 | 0.61 | 0.55 | 0.87 | 0.62 | 0.60 | 0.64 | 0.49 | 0.14 | 4.57 |
| $c_{f,l}$ | 0.23 | 0.27 | 0.23 | 0.22 | 0.40 | 0.28 | 0.41 | 0.22 | 0.28 | 0.32 | 0.23 | 0.25 | 0.20 | 0.85 |
| $\frac{w_l}{P_y}$ | 0.30 | 0.39 | 0.31 | 0.29 | 0.59 | 0.41 | 0.60 | 0.30 | 0.32 | 0.24 | 2.43 | 0.45 | 1.11 | 0.27 |
| $\frac{w_l}{P_f}$ | 0.30 | 0.36 | 0.30 | 0.29 | 0.55 | 0.38 | 0.60 | 0.29 | 0.37 | 0.41 | 0.34 | 0.34 | 0.28 | 1.10 |
| $\frac{w_h}{P_y}$ | 0.80 | 0.94 | 1.16 | 1.25 | 0.83 | 0.92 | 0.82 | 1.23 | 0.72 | 0.48 | 6.69 | 0.89 | 0.78 | 1.55 |
| $\frac{w_h}{P_f}$ | 0.82 | 0.87 | 1.13 | 1.24 | 0.77 | 0.85 | 0.77 | 1.21 | 0.85 | 0.82 | 0.92 | 0.67 | 0.20 | 6.32 |
| $Y$ | 0.11 | 0.37 | 0.15 | 0.13 | 1.94 | 0.33 | 0.22 | 1.15 | 0.28 | 0.19 | 2.79 | 0.38 | 0.64 | 0.46 |
| $F$ | 0.32 | 0.91 | 0.43 | 0.38 | 4.71 | 0.82 | 0.55 | 3.30 | 0.93 | 0.96 | 0.88 | 0.76 | 0.39 | 5.17 |

Table 8: Benchmark vs counterfactual exercises.

Notes: here, the parameter values incorporate variations in the II (-5%) or III (+5%) counterfactual exercise. **POP-line**. *Skilled*: (II): $\Gamma_h^{cc} = 0.99$; (III): $\Gamma_h^{cc} = 1.09$. *Unskilled*: (II): $\Gamma_l^{cc} = 0.96$; (III): $\Gamma_l^{cc} = 1.06$. **TECH-line**. *IT-intensive sector:* (II): $GZ_Y^{cc} = 0.96$; (III): $GZ_Y^{cc} = 1.06$. *Non-IT-intensive sector*: (II): $GZ_F^{cc} = 0.96$; (III): $GZ_F^{cc} = 1.06$.

## 1.5 Conclusion

In this paper, I studied the IG phenomenon theoretically and quantitatively in a closed economy framework. The work was conducted to answer the following research question: *"Can immiserizing growth emerge within an SBTC framework where all agents display the same preferences, and they only differ in their skill level, and consequently in the wage rate earned in the market?"*. I then developed a general equilibrium model with two heterogeneous agents in skills (skilled and unskilled) and two competitive firms (IT-intensive and non-IT intensive). The agents are characterized by identical non-homothetic preferences of the Stone-Geary type, and in the IT-intensive firm, there is the presence of SBTC.

Exploring the model's sensitivity, I contribute to the reference literature by identifying three main drivers capable of driving IG situations: the non-homotheticity, the elasticity of substitution among production inputs, and the existence of SBTC. Specifically, (i) non-homotheticity enables different consumption paths between agents with the same preferences: as income increases, the share of technologically more advanced goods increases; (ii) greater elasticity of substitution between skilled/unskilled in sector $Y$ facilitates substitution between less/more productive workers and leads to the outflow of several skilled agents from sector $F$ to enter into sector $Y$; (iii) SBTC is essential in all IG scenarios: when it is removed, the phenomenon vanishes.

By developing the quantitative section and calibrating the model with U.S. data from EU KLEMS, I find no occurrence of IG because the value of the calibrated parameters does not permit us to leverage the phenomenon's drivers. The estimated model is therefore used to run counterfactual experiments. Studying the contribution of the two annual population growth factors, I find that a decrease in high-skilled agents or an increase in low-skilled ones can generate IG cases. The population composition and the growth rate between skilled/unskilled are other critical elements to consider within the studied phenomenon.

Future model extensions should account for the cross-country dimension and integrate an econometric section. Using data from different countries and comparing them can open a new research line, such as understanding whether it is a geographical phenomenon and finding common factors in the countries considered. Moreover, additional evidence can be provided by evaluating the model's performance using other agent preferences and a broader differentiation in the bundles of goods produced. All these kinds of empirical or theoretical analyses can better validate the phenomenon's mechanisms.

## 1.6 Appendix

The appendix provides the main solving equations for the theoretical model. Further mathematical developments are available upon request to the author of this paper.

## A. Agent's problem

The Lagrangian for agent's maximization problem (P1) is written as:

$$\mathcal{L} = \left[\theta c_{f,a}^{\rho} + (1-\theta)\left(c_{y,a} + \bar{y}\right)^{\rho}\right]^{\frac{1}{\rho}} + \lambda\left[w_a - P_f c_{f,a} - P_y c_{y,a}\right].$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial c_{f,a}} = 0 \quad \Rightarrow \quad C_a^{1-\rho}\theta c_{f,a}^{\rho-1} = \lambda P_f, \tag{I}$$

$$\frac{\partial \mathcal{L}}{\partial c_{y,a}} = 0 \quad \Rightarrow \quad C_a^{1-\rho}(1-\theta)(c_{y,a} + \bar{y})^{\rho-1} = \lambda P_y, \tag{II}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \quad \Rightarrow \quad w_a = P_f c_{f,a} + P_y c_{y,a}, \tag{III}$$

where $C_a$ is the aggregate consumption index defined as follows

$$C_a = \left[\theta c_{f,a}^{\rho} + (1-\theta)(c_{y,a} + \bar{y})^{\rho}\right]^{\frac{1}{\rho}},$$

$$(C_a)^{1-\rho} = \left\{\left[\theta c_{f,a}^{\rho} + (1-\theta)(c_{y,a} + \bar{y})^{\rho}\right]^{\frac{1}{\rho}}\right\}^{1-\rho},$$

$$C_a^{1-\rho} = \left[\theta c_{f,a}^{\rho} + (1-\theta)(c_{y,a} + \bar{y})^{\rho}\right]^{\frac{1-\rho}{\rho}}.$$

The maximized consumption equations resulting from the resolution of the (P1) agent problem are:

$$c_{f,a} = \frac{\left(\frac{w_a}{P_f}\right) + \left(\frac{P_y}{P_f}\right)\bar{y}}{\left(\frac{P_y}{P_f}\right)^{\frac{\rho}{\rho-1}}\left(\frac{1-\theta}{\theta}\right)^{\frac{1}{1-\rho}} + 1}, \tag{1}$$

$$c_{y,a} = \frac{\left(\frac{w_a}{P_f}\right) + \left(\frac{P_y}{P_f}\right)\bar{y}}{\left(\frac{P_y}{P_f}\right)^{\frac{1}{1-\rho}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}} + \left(\frac{P_y}{P_f}\right)} - \bar{y}. \tag{2}$$

## B. Firm's problem

### Y firm

The problem (P2) can be written as:

$$\max_{\{H_y, L_y\}} \left[ P_y Y - w_h H_y - w_l L_y \right].$$

The first-order conditions are:

$$\frac{\partial \Pi_y}{\partial H_y} = 0 \Rightarrow P_y \left[ \mu \left( z_y H_y \right)^{\frac{\sigma_y - 1}{\sigma_y}} + (1-\mu) L_y^{\frac{\sigma_y - 1}{\sigma_y}} \right]^{\frac{1}{\sigma_y - 1}} \mu (z_y)^{\frac{\sigma_y - 1}{\sigma_y}} H_y^{-\frac{1}{\sigma_y}} = w_h, \tag{IV}$$

$$\frac{\partial \Pi_y}{\partial L_y} = 0 \Rightarrow P_y \left[ \mu \left( z_y H_y \right)^{\frac{\sigma_y - 1}{\sigma_y}} + (1-\mu) L_y^{\frac{\sigma_y - 1}{\sigma_y}} \right]^{\frac{1}{\sigma_y - 1}} (1-\mu) L_y^{-\frac{1}{\sigma_y}} = w_l. \tag{V}$$

After a bit of algebra and expressing everything as a function of parameters and relative price, the maximized quantities of high-skilled and low-skilled workers in sector $Y$ are:

$$H_y = \left( \frac{w_l}{P_y} \right)^{-1} \frac{\left\{ H \left[ \frac{\left(\frac{w_h}{P_f}\right) + \left( \frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}} \right) \bar{y}}{\left( \frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}} \right)^{\frac{1}{1-\rho}} \left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}} + \left( \frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}} \right)} - \bar{y} \right] + L \left[ \frac{\left(\frac{w_l}{P_f}\right) + \left( \frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}} \right) \bar{y}}{\left( \frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}} \right)^{\frac{1}{1-\rho}} \left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}} + \left( \frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}} \right)} - \bar{y} \right] \right\}}{\left[ \left( \frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}} \right) + \left( \frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}} \right)^{\sigma_y} \left( \frac{1-\mu}{\mu} \right)^{\sigma_y} \left( \frac{1}{z_y} \right)^{\sigma_y - 1} \right]},$$

$$L_y = \left( \frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}} \right)^{\sigma_y} \left( \frac{1-\mu}{\mu} \right)^{\sigma_y} \left( \frac{1}{z_y} \right)^{\sigma_y - 1} \left[ H_y \right].$$

### F firm

The problem (P3) can be written as:

$$\max_{\{H_f, L_f\}} \left[ P_f F - w_h H_f - w_l L_f \right].$$

The first-order conditions are:

$$\frac{\partial \Pi_f}{\partial H_f} = 0 \Rightarrow P_f \left[ \eta \left( z_f H_f \right)^{\frac{\sigma_f - 1}{\sigma_f}} + (1-\eta) L_f^{\frac{\sigma_f - 1}{\sigma_f}} \right]^{\frac{1}{\sigma_f - 1}} \eta (z_f)^{\frac{\sigma_f - 1}{\sigma_f}} H_f^{-\frac{1}{\sigma_f}} = w_h, \tag{VI}$$

$$\frac{\partial \Pi_f}{\partial L_f} = 0 \Rightarrow P_f \left[ \eta \left( z_f H_f \right)^{\frac{\sigma_f - 1}{\sigma_f}} + (1-\eta) L_f^{\frac{\sigma_f - 1}{\sigma_f}} \right]^{\frac{1}{\sigma_f - 1}} (1-\eta) L_f^{-\frac{1}{\sigma_f}} = w_l. \tag{VII}$$

The maximized quantities of high-skilled and low-skilled workers (function of relative prices and parameters) in sector $F$ are:

$$H_f = \left( \frac{w_l}{P_f} \right)^{-1} \frac{\left\{ H \left[ \frac{\left( \frac{w_h}{P_f} \right) + \left( \frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}} \right) \bar{y}}{\left( \frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}} \right)^{\frac{\rho}{\rho-1}} \left( \frac{1-\theta}{\theta} \right)^{\frac{1}{1-\rho}} + 1} \right] + L \left[ \frac{\left( \frac{w_l}{P_f} \right) + \left( \frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}} \right) \bar{y}}{\left( \frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}} \right)^{\frac{\rho}{\rho-1}} \left( \frac{1-\theta}{\theta} \right)^{\frac{1}{1-\rho}} + 1} \right] \right\}}{\left[ \left( \frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}} \right) + \left( \frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}} \right)^{\sigma_f} \left( \frac{1-\eta}{\eta} \right)^{\sigma_f} \left( \frac{1}{z_f} \right)^{\sigma_f - 1} \right]},$$

$$L_f = \left( \frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}} \right)^{\sigma_f} \left( \frac{1-\eta}{\eta} \right)^{\sigma_f} \left( \frac{1}{z_f} \right)^{\sigma_f - 1} \left[ H_f \right].$$

## C. Matlab equations

I solve my theoretical model on Matlab using a non-linear system of $n$-equations in $m$-unknowns, where $n = m = 4$.

$$
\begin{cases}
\frac{w_l}{P_y} = \dfrac{\left[\mu(z_y)^{\frac{\sigma_y-1}{\sigma_y}} + (1-\mu)\left[\left(\frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}}\right)^{\sigma_y}\left(\frac{1-\mu}{\mu}\right)^{\sigma_y}\left(\frac{1}{z_y}\right)^{\sigma_y-1}\right]^{\frac{\sigma_y-1}{\sigma_y}}\right]^{\frac{\sigma_y}{\sigma_y-1}}}{\left[\left(\frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}}\right)+\left(\frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}}\right)^{\sigma_y}\left(\frac{1-\mu}{\mu}\right)^{\sigma_y}\left(\frac{1}{z_y}\right)^{\sigma_y-1}\right]} \\[2em]
\frac{w_l}{P_f} = \dfrac{\left[\eta(z_f)^{\frac{\sigma_f-1}{\sigma_f}} + (1-\eta)\left[\left(\frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}}\right)^{\sigma_f}\left(\frac{1-\eta}{\eta}\right)^{\sigma_f}\left(\frac{1}{z_f}\right)^{\sigma_f-1}\right]^{\frac{\sigma_f-1}{\sigma_f}}\right]^{\frac{\sigma_f}{\sigma_f-1}}}{\left[\left(\frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}}\right)+\left(\frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}}\right)^{\sigma_f}\left(\frac{1-\eta}{\eta}\right)^{\sigma_f}\left(\frac{1}{z_f}\right)^{\sigma_f-1}\right]} \\[2em]
L = \left(\frac{w_l}{P_f}\right)^{-1}\dfrac{\left\{H\left[\dfrac{\left(\frac{w_h}{P_f}\right)+\left(\frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}}\right)\bar{y}}{\left(\frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}}\right)^{\frac{\rho}{\rho-1}}\left(\frac{1-\theta}{\theta}\right)^{\frac{1}{1-\rho}}+1}\right]+L\left[\dfrac{\left(\frac{w_l}{P_f}\right)+\left(\frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}}\right)\bar{y}}{\left(\frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}}\right)^{\frac{\rho}{\rho-1}}\left(\frac{1-\theta}{\theta}\right)^{\frac{1}{1-\rho}}+1}\right]\right\}}{\left[\left(\frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}}\right)^{1-\sigma_f}\left(\frac{\eta}{1-\eta}\right)^{\sigma_f}(z_f)^{\sigma_f-1}+1\right]} + \left(\frac{w_l}{P_y}\right)^{-1}\dfrac{\left\{H\left[\dfrac{\left(\frac{w_h}{P_f}\right)+\left(\frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}}\right)\bar{y}}{\left(\frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}}\right)^{\frac{1}{1-\rho}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}}+\left(\frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}}\right)}-\bar{y}\right]+L\left[\dfrac{\left(\frac{w_l}{P_f}\right)+\left(\frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}}\right)\bar{y}}{\left(\frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}}\right)^{\frac{1}{1-\rho}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}}+\left(\frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}}\right)}-\bar{y}\right]\right\}}{\left[\left(\frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}}\right)^{1-\sigma_y}\left(\frac{\mu}{1-\mu}\right)^{\sigma_y}(z_y)^{\sigma_y-1}+1\right]} \\[2em]
H = \left(\frac{w_l}{P_f}\right)^{-1}\dfrac{\left\{H\left[\dfrac{\left(\frac{w_h}{P_f}\right)+\left(\frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}}\right)\bar{y}}{\left(\frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}}\right)^{\frac{\rho}{\rho-1}}\left(\frac{1-\theta}{\theta}\right)^{\frac{1}{1-\rho}}+1}\right]+L\left[\dfrac{\left(\frac{w_l}{P_f}\right)+\left(\frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}}\right)\bar{y}}{\left(\frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}}\right)^{\frac{\rho}{\rho-1}}\left(\frac{1-\theta}{\theta}\right)^{\frac{1}{1-\rho}}+1}\right]\right\}}{\left[\left(\frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}}\right)+\left(\frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}}\right)^{\sigma_f}\left(\frac{1-\eta}{\eta}\right)^{\sigma_f}\left(\frac{1}{z_f}\right)^{\sigma_f-1}\right]} + \left(\frac{w_l}{P_y}\right)^{-1}\dfrac{\left\{H\left[\dfrac{\left(\frac{w_h}{P_f}\right)+\left(\frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}}\right)\bar{y}}{\left(\frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}}\right)^{\frac{1}{1-\rho}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}}+\left(\frac{\frac{w_h}{P_f}}{\frac{w_h}{P_y}}\right)}-\bar{y}\right]+L\left[\dfrac{\left(\frac{w_l}{P_f}\right)+\left(\frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}}\right)\bar{y}}{\left(\frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}}\right)^{\frac{1}{1-\rho}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}}+\left(\frac{\frac{w_l}{P_f}}{\frac{w_l}{P_y}}\right)}-\bar{y}\right]\right\}}{\left[\left(\frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}}\right)+\left(\frac{\frac{w_h}{P_f}}{\frac{w_l}{P_f}}\right)^{\sigma_y}\left(\frac{1-\mu}{\mu}\right)^{\sigma_y}\left(\frac{1}{z_y}\right)^{\sigma_y-1}\right]}
\end{cases}
$$

Figure 4: Non-linear system of $n$-equations in $m$-unknowns.

By defining the relative prices as the four unknowns of the model

$$\frac{w_l}{P_y} = X(1); \qquad \frac{w_l}{P_f} = X(2); \qquad \frac{w_h}{P_y} = X(3); \qquad \frac{w_h}{P_f} = X(4);$$

and equating the formulas to zero, the non-linear system can be expressed as in Fig. 5:

$$
\begin{cases}
F(1) = X(1) - \dfrac{\left[\mu (z_y)^{\frac{\sigma_y - 1}{\sigma_y}} + (1-\mu)\left[\left(\frac{X(4)}{X(2)}\right)^{\sigma_y}\left(\frac{1-\mu}{\mu}\right)^{\sigma_y}\left(\frac{1}{z_y}\right)^{\sigma_y - 1}\right]^{\frac{\sigma_y - 1}{\sigma_y}}\right]^{\frac{\sigma_y}{\sigma_y - 1}}}{\left[\left(\frac{X(4)}{X(2)}\right) + \left(\frac{X(4)}{X(2)}\right)^{\sigma_y}\left(\frac{1-\mu}{\mu}\right)^{\sigma_y}\left(\frac{1}{z_y}\right)^{\sigma_y - 1}\right]} \\[2ex]
F(2) = X(2) - \dfrac{\left[\eta (z_f)^{\frac{\sigma_f - 1}{\sigma_f}} + (1-\eta)\left[\left(\frac{X(4)}{X(2)}\right)^{\sigma_f}\left(\frac{1-\eta}{\eta}\right)^{\sigma_f}\left(\frac{1}{z_f}\right)^{\sigma_f - 1}\right]^{\frac{\sigma_f - 1}{\sigma_f}}\right]^{\frac{\sigma_f}{\sigma_f - 1}}}{\left[\left(\frac{X(4)}{X(2)}\right) + \left(\frac{X(4)}{X(2)}\right)^{\sigma_f}\left(\frac{1-\eta}{\eta}\right)^{\sigma_f}\left(\frac{1}{z_f}\right)^{\sigma_f - 1}\right]} \\[2ex]
F(3) = L - (X(2))^{-1} \dfrac{\left\{H\left[\frac{X(4)+\left(\frac{X(4)}{X(3)}\right)\bar{y}}{\left(\frac{X(4)}{X(3)}\right)^{\frac{\rho}{\rho-1}}\left(\frac{1-\theta}{\theta}\right)^{\frac{1}{1-\rho}}+1}\right] + L\left[\frac{X(2)+\left(\frac{X(2)}{X(1)}\right)\bar{y}}{\left(\frac{X(2)}{X(1)}\right)^{\frac{\rho}{\rho-1}}\left(\frac{1-\theta}{\theta}\right)^{\frac{1}{1-\rho}}+1}\right]\right\}}{\left[\left(\frac{X(4)}{X(2)}\right)^{1-\sigma_f}\left(\frac{\eta}{1-\eta}\right)^{\sigma_f}(z_f)^{\sigma_f - 1}+1\right]} - (X(1))^{-1} \dfrac{\left\{H\left[\frac{X(4)+\left(\frac{X(4)}{X(3)}\right)\bar{y}}{\left(\frac{X(4)}{X(3)}\right)^{\frac{1}{1-\rho}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}}+\left(\frac{X(4)}{X(3)}\right)} - \bar{y}\right] + L\left[\frac{X(2)+\left(\frac{X(2)}{X(1)}\right)\bar{y}}{\left(\frac{X(2)}{X(1)}\right)^{\frac{1}{1-\rho}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}}+\left(\frac{X(2)}{X(1)}\right)} - \bar{y}\right]\right\}}{\left[\left(\frac{X(4)}{X(2)}\right)^{1-\sigma_y}\left(\frac{\mu}{1-\mu}\right)^{\sigma_y}(z_y)^{\sigma_y - 1}+1\right]} \\[2ex]
F(4) = H - (X(2))^{-1} \dfrac{\left\{H\left[\frac{X(4)+\left(\frac{X(4)}{X(3)}\right)\bar{y}}{\left(\frac{X(4)}{X(3)}\right)^{\frac{\rho}{\rho-1}}\left(\frac{1-\theta}{\theta}\right)^{\frac{1}{1-\rho}}+1}\right] + L\left[\frac{X(2)+\left(\frac{X(2)}{X(1)}\right)\bar{y}}{\left(\frac{X(2)}{X(1)}\right)^{\frac{\rho}{\rho-1}}\left(\frac{1-\theta}{\theta}\right)^{\frac{1}{1-\rho}}+1}\right]\right\}}{\left[\left(\frac{X(4)}{X(2)}\right)+\left(\frac{X(4)}{X(2)}\right)^{\sigma_f}\left(\frac{1-\eta}{\eta}\right)^{\sigma_f}\left(\frac{1}{z_f}\right)^{\sigma_f - 1}\right]} - (X(1))^{-1} \dfrac{\left\{H\left[\frac{X(4)+\left(\frac{X(4)}{X(3)}\right)\bar{y}}{\left(\frac{X(4)}{X(3)}\right)^{\frac{1}{1-\rho}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}}+\left(\frac{X(4)}{X(3)}\right)} - \bar{y}\right] + L\left[\frac{X(2)+\left(\frac{X(2)}{X(1)}\right)\bar{y}}{\left(\frac{X(2)}{X(1)}\right)^{\frac{1}{1-\rho}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}}+\left(\frac{X(2)}{X(1)}\right)} - \bar{y}\right]\right\}}{\left[\left(\frac{X(4)}{X(2)}\right)+\left(\frac{X(4)}{X(2)}\right)^{\sigma_y}\left(\frac{1-\mu}{\mu}\right)^{\sigma_y}\left(\frac{1}{z_y}\right)^{\sigma_y - 1}\right]}
\end{cases}
$$

Figure 5: Non-linear system expressed as four functions $\{F(1), ..., F(4)\}$ on Matlab.

## D. Preferences and special cases

**Special case (i) : from Stone-Geary to Murphy (2016) general setting preferences**

I rewrite my utility function to distinguish the parameter $\theta$ by agent type

$$U_a = \left[\theta_a c_{f,a}^{\rho} + (1-\theta_a)\left(c_{y,a} + \bar{y}\right)^{\rho}\right]^{\frac{1}{\rho}}.$$

Depending on the value assigned to the parameter $\theta_a$, it is possible to obtain as a special case the Murphy (2016) preferences.

**Proposition 1:** if $\theta_h = 0$ and $\theta_l = 1$ $\Rightarrow$ agents consume only one type of good: Murphy (2016).

**Proof of proposition 1:** by setting $\theta_h = 0$ in the skilled agent function and with a bit of algebra:

$$U_h = \left[\theta_h c_{f,h}^{\rho} + (1-\theta_h)\left(c_{y,h} + \bar{y}\right)^{\rho}\right]^{\frac{1}{\rho}},$$

$$U_h^{\rho} = 0 \cdot c_{f,h}^{\rho} + (1-0)\left(c_{y,h} + \bar{y}\right)^{\rho},$$

$$U_h^{\rho} = \left(c_{y,h} + \bar{y}\right)^{\rho},$$

$$U_h = c_{y,h} + \bar{y}.$$

the skilled agent consumes only one type of good ($c_y$). The non-homothetic element $\bar{y}$ does not change the meaning and can either be kept or set to zero (depending on preference type). Now, by setting $\theta_l = 1$ in the unskilled agent function:

$$U_l = \left[\theta_l c_{f,l}^{\rho} + (1-\theta_l)\left(c_{y,l} + \bar{y}\right)^{\rho}\right]^{\frac{1}{\rho}},$$

$$U_l^{\rho} = 1 \cdot c_{f,l}^{\rho} + (1-1)\left(c_{y,l} + \bar{y}\right)^{\rho},$$

$$U_l^{\rho} = c_{f,l}^{\rho},$$

$$U_l = c_{f,l}.$$

the unskilled agent consumes only one type of good ($c_f$). Starting from my Stone-Geary preferences, distinguishing the $\theta$ parameter for agents and setting $\theta_h = 0$ and $\theta_l = 1$, I obtained as a special case the Murphy (2016) preferences, in which each type of agent consumes only one bundle of goods. ■

**Special case (ii): from Stone-Geary to CES and from CES to Cobb-Douglas preferences**

By imposing the non-homotheticity parameter $\bar{y} = 0$, the preferences change from Stone-Geary to CES. Thus, when $\rho \to 0$, CES becomes a Cobb-Douglas. In these terms, I can state that Cobb-Douglas preferences are a special case of CES preferences.

**Proposition 2:** when $\rho \to 0$, CES becomes Cobb-Douglas.

**Proof of proposition 2:** I set $\bar{y} = 0$ and transform non-homothetic preferences into CES preferences.

$$U = \left[\theta c_{f,a}^{\rho} + (1-\theta)\left(c_{y,a} + \bar{y}\right)^{\rho}\right]^{\frac{1}{\rho}},$$

$$U = \left[\theta c_{f,a}^{\rho} + (1-\theta)\left(c_{y,a}\right)^{\rho}\right]^{\frac{1}{\rho}},$$

Applying the logarithm to both members of the CES preferences

$$ln\ U = ln\left[\theta c_{f,a}^{\rho} + (1-\theta)\left(c_{y,a}\right)^{\rho}\right]^{\frac{1}{\rho}},$$

$$ln\ U = \frac{ln\left[\theta c_{f,a}^{\rho} + (1-\theta)\left(c_{y,a}\right)^{\rho}\right]}{\rho}.$$

I want to study the right limit to check what occurs when $\rho \to 0$:

$$\lim_{\rho\to 0} ln\ U = \lim_{\rho\to 0} \frac{ln\left[\theta c_{f,a}^{\rho} + (1-\theta)\left(c_{y,a}\right)^{\rho}\right]}{\rho}.$$

So, for simplicity I define

$$m(\rho) = ln\left[\theta c_{f,a}^{\rho} + (1-\theta)\left(c_{y,a}\right)^{\rho}\right],$$

and

$$n(\rho) = \rho,$$

to apply the L'Hôpital's rule:

$$\lim_{\rho \to 0} \frac{m(\rho)}{n(\rho)} = \lim_{\rho \to 0} \frac{m'(\rho)}{n'(\rho)} = \lim_{\rho \to 0} \frac{\theta c_{f,a}^{\rho} ln\,(c_{f,a}) + (1-\theta) c_{y,a}^{\rho} ln\,(c_{y,a})}{\theta c_{f,a}^{\rho} + (1-\theta) c_{y,a}^{\rho}},$$

$$\lim_{\rho \to 0} \frac{m'(\rho)}{n'(\rho)} = \lim_{\rho \to 0} \frac{\theta c_{f,a}^{0} ln\,(c_{f,a}) + (1-\theta) c_{y,a}^{0} ln\,(c_{y,a})}{\theta c_{f,a}^{0} + (1-\theta) c_{y,a}^{0}},$$

$$\lim_{\rho \to 0} \frac{m'(\rho)}{n'(\rho)} = \theta ln\, c_{f,a} + (1-\theta) ln\, c_{y,a}.$$

Therefore,

$$\lim_{\rho \to 0} ln\, U = \theta ln\, c_{f,a} + (1-\theta) ln\, c_{y,a} = ln\, c_{f,a}^{\theta} + ln\, c_{y,a}^{1-\theta},$$

$$ln\, U = ln\, c_{f,a}^{\theta} c_{y,a}^{1-\theta}, \qquad \exp^{ln\, U} = \exp^{ln\, c_{f,a}^{\theta} c_{y,a}^{1-\theta}}, \qquad U = c_{f,a}^{\theta} c_{y,a}^{1-\theta},$$

where the parameter $\theta$ plays the same role as the classical Cobb-Douglas $\alpha$ parameter. So, rewriting the last equation, I get:

$$U = c_{f,a}^{\alpha} c_{y,a}^{1-\alpha} \quad \blacksquare$$

## E. Consumption shares

I introduce a generalized version of the agent problem with Cobb-Douglas preferences present in Murphy (2016); I then compare the consumption shares emerging from his model with the consumption shares in my model.

### Agent's problem with a Cobb-Douglas setting

The agent solves the following maximization problem:

$$\max_{\{c_{f,a},\, c_{y,a}\}} U_a = c_{f,a}^{\delta} c_{y,a}^{1-\delta}$$

subject to

$$P_f c_{f,a} + P_y c_{y,a} = w_a \ ,$$

where $\delta = \{\alpha, \beta\}$, $\alpha$ is the CD parameter for the unskilled agent, $\beta$ is the parameter for the skilled agent. The Lagrangian for agent's maximization problem is written as

$$\mathcal{L} = \left[c_{f,a}^{\delta} c_{y,a}^{1-\delta}\right] + \lambda \left[w_a - P_f c_{f,a} - P_y c_{y,a}\right],$$

and the first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial c_{f,a}} = 0 \quad \Rightarrow \quad \delta c_{f,a}^{\delta-1} c_{y,a}^{1-\delta} = \lambda P_f,$$

$$\frac{\partial \mathcal{L}}{\partial c_{y,a}} = 0 \quad \Rightarrow \quad (1-\delta)\, c_{f,a}^{\delta} c_{y,a}^{-\delta} = \lambda P_y,$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \quad \Rightarrow \quad w_a = P_f c_{f,a} + P_y c_{y,a}.$$

The maximized consumption, in the specific case, for both agents are:

$$c_{f,h} = \frac{w_h}{P_f}\beta, \qquad c_{y,h} = \frac{w_h}{P_y}(1-\beta),$$

$$c_{f,l} = \frac{w_l}{P_f}\alpha, \qquad c_{y,l} = \frac{w_l}{P_y}(1-\alpha).$$

**Consumption shares with Cobb-Douglas preferences**

Rearranging the last equations, I can conduct two concatenated interpretations:

$$P_f c_{f,h} = w_h \beta, \qquad P_y c_{y,h} = w_h (1-\beta),$$

$$P_f c_{f,l} = w_l \alpha, \qquad P_y c_{y,l} = w_l (1-\alpha),$$

(i) the consumption value is a constant fraction of the income and,

$$Share^{CD}_{f,skilled} = \frac{P_f c_{f,h}}{w_h} = \beta, \qquad Share^{CD}_{y,skilled} = \frac{P_y c_{y,h}}{w_h} = (1-\beta),$$

$$Share^{CD}_{f,unskilled} = \frac{P_f c_{f,l}}{w_l} = \alpha, \qquad Share^{CD}_{y,unskilled} = \frac{P_y c_{y,l}}{w_l} = (1-\alpha),$$

(ii) the consumption share is fixed.

The Cobb-Douglas version of the model is characterized by consumption shares determined from the $\beta$ ($\alpha$) parameter for the skilled (unskilled) agent.

**Consumption shares with Stone-Geary preferences**

The nominal consumption shares, in my model with non-homothetic preferences, are designed as follows:

$$Share^{SG}_{y,skilled} = \frac{P_y c_{y,h}}{P_y c_{y,h} + P_f c_{f,h}} = \frac{\frac{P_y c_{y,h}}{P_y}}{\frac{P_y c_{y,h} + P_f c_{f,h}}{P_y}} = \frac{c_{y,h}}{c_{y,h} + \frac{\frac{w_h}{P_y}}{\frac{w_h}{P_f}} c_{f,h}},$$

$$Share^{SG}_{f,skilled} = 1 - Share^{SG}_{y,skilled},$$

$$Share^{SG}_{y,unskilled} = \frac{P_y c_{y,l}}{P_y c_{y,l} + P_f c_{f,l}} = \frac{\frac{P_y c_{y,l}}{P_y}}{\frac{P_y c_{y,l} + P_f c_{f,l}}{P_y}} = \frac{c_{y,l}}{c_{y,p} + \frac{\frac{w_l}{P_y}}{\frac{w_l}{P_f}} c_{f,l}},$$

$$Share^{SG}_{f,unskilled} = 1 - Share^{SG}_{y,unskilled},$$

where I remember that in terms of relative price

$$\frac{\frac{w_{a=\{h,l\}}}{P_y}}{\frac{w_{a=\{h,l\}}}{P_f}} = \frac{w_a}{P_y}\frac{P_f}{w_a} = \frac{P_f}{P_y}.$$

The main difference in the shares is that in the CD preference model, the shares are fixed, i.e., determined by the preferences' $\alpha$ or $\beta$ parameter. In contrast, the shares in the SG model depend jointly on wages $(w_r, w_p)$ and goods prices $(P_y, P_f)$. As relative prices change, the shares incorporate these variations and allow the agent to adjust the composition of the purchased bundle. In other words, the nominal shares that originate from non-homothetic preferences allow, as income increases, different proportions of consumption.

## F. EU KLEMS

The EU KLEMS dataset consists of 34 industries plus 8 aggregates. All industries in the data are listed as follows: (1) Agriculture, forestry and fishing; (2) Mining and quarrying; (3) Food products, beverages and tobacco; (4) Textiles, wearing apparel, leather and related products; (5) Wood and paper products; printing and reproduction of recorded media; (6) Coke and refined petroleum products; (7) Chemicals and chemical products; (8) Rubber and plastics products, and other non-metallic mineral products; (9) Basic metals and fabricated metal products, except machinery and equipment; (10) Electrical and optical equipment; (11) Machinery and equipment n.e.c.; (12) Transport equipment; (13) Other manufacturing; repair and installation of machinery and equipment; (14) Electricity, gas and water supply; (15) Construction; (16) Wholesale and retail trade and repair of motor vehicles and motorcycles; (17) Wholesale trade, except of motor vehicles and motorcycles; (18) Retail trade, except of motor vehicles and motorcycles; (19) Transport and storage; (20) Postal and courier activities; (21) Accomodation and food service activities; (22) Publishing, audiovisual and broadcasting activities; (23) Telecommunications; (24) IT and other information services; (25) Financial and insurance activities; (26) Real estate activities; (27) Professional, scientific, technical, administrative and support service activities; (28) Public administration and defence; compulsory social security; (29) Education; (30) Health and social work; (31) Arts, entertainment and recreation; (32) Other service activities; (33) Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use; (34) Activities of extraterritorial organizations and bodies.

Aggregates are: Total industries; Market economy; Total manufacturing; Wholesale and retail trade; repair of motor vehicles and motorcycles; Transportation and storage; Information and communication; Community social and personal services; Arts, entertainment, recreation and other service activities.

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