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State-dependent leverage in rare disaster models without time-varying disaster probability

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State-dependent leverage in rare disaster models without time-varying disaster probability

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Abstract

Rare disaster models match the equity premium and the risk-free rate but understate the volatility of equity returns: in Barro and Jin (2021) the standard deviation of the levered return is 0.0975, against 0.245 in the data. Levered returns there are built from a constant debt-equity ratio, as is standard in this literature, which implies that firms offset the effect of price movements on market leverage, contrary to what the capital structure literature finds. I replace it with debt fixed in units of consumption, so that leverage rises when valuations fall, holding mean leverage fixed. The standard deviation of the levered excess return rises by 1.93% and the Sharpe ratio falls by 1.44%, while the mean premium rises by 0.47%: leverage is highest where valuations are low, and those are the states where the excess return is most dispersed. The effect is small for a reason internal to the model, since leverage moves only as much as the valuation ratio, which the model itself moves little.

Keywords: rare disasters, equity premium, equity volatility, corporate leverage
JEL classification: G12, G32, E21, E44

1 Introduction

Consumption-based asset pricing has made considerable progress over the last four decades, but several problems remain open. One of the most persistent is the difficulty these models have with the second moments of equity returns: the volatility of the equity premium and, with it, the Sharpe ratio. In the calibration of Barro and Jin (2021) the standard deviation of the levered equity return is 0.0975 against 0.245 in the data, and the Sharpe ratio is 0.82 against 0.295. Their model combines the long-run risk and stochastic volatility components of Bansal and Yaron (2004) with the rare disaster framework pioneered by

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Rietz (1988) and developed by Barro (2006), and prices assets under the recursive preferences of Epstein and Zin (1989) and Weil (1990). The authors identify the omission of a time-varying disaster probability as the major remaining gap, and describe that extension as an order of magnitude harder numerically. This paper asks a narrower question: how much of the gap is due to an auxiliary assumption the model makes about corporate financing rather than about the disaster process.

Equity in these models is a levered claim on aggregate consumption, and the levered return is constructed from the unlevered one with a debt-equity ratio held constant at one half, the Flow of Funds average for US nonfinancial corporations (Barro, 2006), adopted by Barro and Jin (2021) following Nakamura et al. (2013). The value is calibrated on that historical average, but holding it constant, beyond being a simplification, implicitly assumes that debt moves with the value of equity period by period, and in exactly the proportion that keeps the ratio at one half. The evidence on capital structure points the other way at annual frequency. Welch (2004) finds that US firms do not counteract the mechanical effect of stock returns on their debt-equity ratios, so that returns account for about 40% of the dynamics of the ratio, and Yin and Ritter (2020) show that once the passive effect of prices is removed, market leverage adjusts toward its target at about 10% per year; the survey evidence in Graham and Harvey (2001) points the same way. What debt does track is cash flow: Lian and Ma (2021) document that about 80% of US nonfinancial corporate debt by value rests on operating earnings rather than on asset values, and Drechsel (2023) studies the macroeconomic implications of earnings-based constraints. The main dissent is Flannery and Rangan (2006), who estimate that firms close about a third of the distance to their target each year, a speed that Yin and Ritter (2020) attribute in large part to the passive component of price movements.

Following the evidence that debt is tied to cash flow, I assume instead that debt is fixed in units of consumption, that is, a constant multiple of the current flow. The debt-equity ratio then inherits the variation of the price-dividend ratio of the unlevered claim on consumption, and leverage is high exactly where the price of that claim is low. The comparison is mean-preserving and introduces no free parameter. Nothing else changes: the same code, the same calibration and the same simulated path of states. Since leverage does not enter the Euler equation, the fixed point that determines the price-dividend ratio of the consumption claim is identical in the two cases, and the difference between the two sets of moments is attributable to the leverage assumption alone.

The effect goes in the direction the mechanism implies and is small. The standard deviation of the levered excess return rises by 1.93%, the mean premium rises by 0.47%, and the Sharpe ratio falls by 1.44%. Leverage is highest where the valuation ratio is low, and in this model those are the states where the excess return is both larger in mean and more dispersed, so both moments of the levered return rise, the dispersion by more than the mean. Replacing the discrete admissibility rule with its continuous counterpart changes the moments by 0.04%, so the result comes from the leverage specification and not from the treatment of the constraint.

The size of the effect has a simple source. Leverage here is a function of the valuation ratio alone, so its dispersion is the dispersion of the price-dividend ratio, which in this model is small: the coefficient of variation of leverage is 0.12. The conclusion is negative and quantified. Replacing the constant debt-equity ratio with a specification the evidence supports leaves the second moments of the model close to where they were, so the variability of valuations has to be addressed first, and only then can a leverage assumption of this kind produce an appreciable effect.

2 Leverage in the Barro–Jin model

I take the model of Barro and Jin (2021) and its calibration as given, and recall here only what the leverage assumption rests on; the model is set out in full in their paper. A representative agent with recursive preferences (Epstein and Zin, 1989; Weil, 1990) prices a claim to aggregate consumption. Log consumption growth combines a normal shock, a persistent growth component and stochastic volatility with rare disasters, which have a permanent and a transitory part and arrive according to a Markov structure defined at the world and at the country level. The state s_t collects the two disaster indicators, the transitory disaster component, the volatility state and the growth component. The price-dividend ratio of the unlevered claim, $PDR(s_t)$, solves a recursion in the state and is computed on a discretised state space; moments are then obtained from a long simulated path.

The dividend of the unlevered claim is consumption itself, so its ex-dividend value is $P_t = PDR(s_t)\, C_t$ and its gross return between t and $t+1$ is

$$R^u_{t+1} \;=\; \frac{PDR(s_{t+1})+1}{PDR(s_t)} \cdot \frac{C_{t+1}}{C_t}. \tag{1}$$

The gross return on a one-period riskless bill is R^f. Both are functions of the state, and neither depends on how the claim is financed. Lower-case letters denote the corresponding net returns, $r = R - 1$.

Equity is a levered claim on the same cash flow. Under Modigliani and Miller (1958) the value of the unlevered claim splits into debt and equity, $P_t = D_t + E_t$, and the debt-equity ratio is $\varsigma_t = D_t/E_t$. With debt that is riskless over the period, the levered return and the levered excess return are

$$R^e_{t+1} = (1+\varsigma_t)\, R^u_{t+1} - \varsigma_t R^f_{t+1}, \qquad R^e_{t+1} - R^f_{t+1} = (1+\varsigma_t)\left(R^u_{t+1} - R^f_{t+1}\right), \tag{2}$$

so the whole effect of financing on returns runs through ς_t. The cash flow left to equity holders net of interest, expressed as a yield, is

$$DY(s_t) \;=\; \frac{1+\varsigma_t}{PDR(s_t)} \;-\; \varsigma_t\left(R^f(s_t)-1\right), \tag{3}$$

and admissibility requires it to be positive: where it is not, Barro and Jin (2021) lower ς_t until it is. In their calibration ς_t is one half wherever that constraint does not bind.

Two features of this construction matter for what follows. Leverage does not enter the Euler equation, so $PDR(s)$, $R^f(s)$ and R^u are unaffected by the leverage assumption and the fixed point need not be solved again. And ς_t is chosen at the beginning of the period, before the return in (2) is realised.

3 Debt fixed in units of consumption

I keep everything in Section 2 and change the assumption on debt alone. Debt is a constant multiple of the current flow,

$$D_t = \bar{d}\,C_t, \tag{4}$$

so that equity is worth $E_t = P_t - D_t = [PDR(s_t) - \bar{d}]\,C_t$, positive wherever $PDR(s_t) > \bar{d}$, and the debt-equity ratio becomes

$$\varsigma(s_t) \;=\; \frac{\bar{d}}{PDR(s_t)-\bar{d}}. \tag{5}$$

Leverage is now a function of the state, and it depends on the state only through the valuation ratio: it is high where the unlevered claim is cheap and low where it is expensive. Its elasticity to the price of the claim is $-(1+\varsigma)$, so the same proportional fall in prices raises leverage by more the more levered the claim already is.

The two assumptions differ in what debt is tied to. A constant ratio ties debt to the market value of equity; (4) ties it to the cash flow the claim pays. The wedge between them is the valuation ratio, and only that. Because debt is proportional to consumption, it falls with consumption in a disaster exactly as it rises with consumption in normal times, so leverage does not respond to the size of the cash flow shock and what (5) isolates is the response of leverage to valuations.

Tying debt to the flow rather than to market value is the polar case of the evidence recalled in Section 1: firms do not undo the effect of prices on market leverage (Welch, 2004; Yin and Ritter, 2020), while debt capacity is written on operating earnings (Lian and Ma, 2021; Drechsel, 2023). In structural models of financing the same pattern arises at the aggregate level, where infrequent refinancing leaves market leverage moving against valuations (Bhamra et al., 2010).

The constant $\bar{d}$ is not free. It is set so that the average of $\varsigma(s)$ along the simulated path equals average leverage in the baseline, where the ratio is one half in every state except where admissibility binds. The comparison is therefore mean-preserving: the two specifications differ in the dispersion of leverage and in where leverage is high, not in its level. Admissibility is imposed as in Barro and Jin (2021), by lowering ς in steps where (3) would be negative. Section 4 also reports the results under a continuous counterpart of that rule,

$$\varsigma(s) = \min\left\{\frac{\bar{d}}{PDR(s) - \bar{d}},\ \varsigma_{\max}(s)\right\}, \qquad \varsigma_{\max}(s) = \frac{1}{PDR(s)\left(R^f(s) - 1\right) - 1}, \tag{6}$$

which bites where the denominator of $\varsigma_{\max}$ is positive and leaves leverage unrestricted otherwise. Either way the constraint is rarely active, on 0.20% of the simulated years, and the two rules give almost identical moments.

4 Results

Table 1 reports the moments of the levered return in the baseline and under debt fixed in units of consumption. Average leverage is the same across specifications by construction, and the risk-free rate is untouched, since the leverage assumption plays no part in its pricing.

The direction is the one the mechanism predicts. The standard deviation of the levered excess return rises from 0.0873 to 0.0890, an increase of 1.93%, and that of the levered return from 0.0975 to 0.0987. The mean excess return rises by 0.47%, so in relative terms volatility moves about four times as much as the premium, and the Sharpe ratio falls by 1.44%, from 0.8201 to 0.8083. Set against the distance from the data, the magnitudes are small: the rise in the volatility of the excess return covers about 1% of the distance to the observed 0.245, and the fall in the Sharpe ratio about 2% of the distance to the observed 0.295.

What produces the change is the dispersion of leverage rather than the treatment of the constraint. The coefficient of variation of leverage goes from 0.0195 in the baseline, where the only variation comes from the few states in which admissibility binds, to 0.1203 under (5). Replacing the step rule with the continuous rule in (6) moves every moment in the table by at most 0.04%, about one fiftieth of the effect just described.

Table 1: Return moments under alternative leverage specifications

	$E[r^e]$	sd(r^e)	$E[r^e - r^f]$	sd$(r^e - r^f)$	Sharpe	$CV(\varsigma)$
Panel A. Levels						
Data	0.0790	0.2450	0.0715	0.2450	0.2950	–
(1) Barro–Jin, $\varsigma = 0.5$	0.0794	0.0975	0.0716	0.0873	0.8201	0.0195
(2) $\varsigma(s)$, step constraint	0.0797	0.0987	0.0720	0.0890	0.8083	0.1203
(3) $\varsigma(s)$, continuous constraint	0.0797	0.0988	0.0720	0.0891	0.8082	0.1199
Panel B. Percentage changes						
(2) relative to (1)		+1.27	+0.47	+1.93	−1.44	
(3) relative to (2)		+0.03	+0.01	+0.04	−0.02	
(3) relative to (1)		+1.31	+0.49	+1.97	−1.45	

Notes. Moments of gross annual returns along a simulated path of 10^6 years, computed with the model, the parameter estimates and the simulated states of Barro and Jin (2021); r^e is the return on levered equity and r^f the risk-free rate. The row *Data* reproduces column 1 of their Table 4, long-term averages for the 17 countries with long-term data on asset returns; row (1) reproduces their column 2 and is the baseline of the comparison. Rows (2) and (3) replace the constant debt-equity ratio with debt fixed in units of consumption, (4), under the step and the continuous admissibility rule respectively. Average leverage is 0.4996 in all three specifications, so the comparison is mean-preserving; the implied $\bar{d}$ is 9.4772 under the step rule and 9.4765 under the continuous one. The risk-free rate does not depend on the leverage assumption and is the same throughout, with $E[r^f] = 0.0078$ and $\mathrm{sd}(r^f) = 0.0253$. The admissibility constraint is active on 0.27% of the simulated years in (1) and on 0.20% in (2) and (3). $CV(\varsigma)$ is the coefficient of variation of leverage along the path. Panel B reports percentage changes in the corresponding entries of Panel A.

5 Mechanism and magnitude

Under (5) leverage is a function of the valuation ratio alone, so whatever the model does to valuations it does to leverage as well. Table 2 groups the simulated years by the volatility state of consumption growth and reports, for each group, the average valuation ratio, the average debt-equity ratio, and the first two moments of the unlevered excess return. Valuations fall as uncertainty rises, from 30.5 to 25.7, and since debt does not follow the price, leverage rises from 0.45 to 0.59.

The last two columns show what the multiplier $(1+\varsigma)$ in (2) meets in those states. The unlevered excess return is both larger and more dispersed where uncertainty is higher, so leverage is high where both moments are high, and both moments of the levered return rise when the constant ratio is replaced. They do not rise by the same amount. With mean leverage held fixed, the premium responds only to the covariance between leverage and the conditional mean of the excess return, while the dispersion responds to the covariance with the conditional second moment and to the dispersion of leverage itself.[1] In Table 1 the first is worth 0.47% and the second 1.93%, and the Sharpe ratio falls.

[1]Since leverage is measurable at the beginning of the period, the variance of the levered excess return splits exactly into $\mathrm{Var}[(1+\varsigma)m(s)]$ and $E[(1+\varsigma)^2 v(s)]$, where $m(s)$ and $v(s)$ are the mean and the variance of the unlevered excess return conditional on the state. The second of these accounts for 68.2% of the increase and does not enter the mean at all. Along the simulated path leverage correlates 0.50 with $v(s)$ and 0.47 with the volatility state.

Table 2: Valuations, leverage and the unlevered excess return across volatility states

Mean PDR	Mean ς	$E[R^u - R^f]$	sd$(R^u - R^f)$	Share of years
30.52	0.453	3.22%	4.89%	9.50%
29.49	0.477	4.13%	5.36%	30.15%
28.44	0.504	4.97%	5.86%	36.90%
27.43	0.534	5.77%	6.33%	18.94%
26.50	0.564	6.61%	6.80%	4.17%
25.73	0.592	7.46%	6.98%	0.33%

Notes. Averages along the simulated path of Table 1, under debt fixed in units of consumption with the step rule. Each row is one volatility state of the model, from the least to the most volatile: the standard deviation of consumption growth implied by these states runs from 0.71% to 3.94% per year. Two further states are visited, with a combined weight below 0.05% of the years. The last two columns are the mean and the standard deviation of the unlevered excess return in the years that start in each state, so they are the quantities the multiplier $(1 + \varsigma)$ acts on. The third column averaged across the rows, weighted by the last one, gives the unlevered counterpart of the mean premium in row (1) of Table 1; the fourth does not average that way, since the dispersion of the excess return also reflects the differences between states. Leverage is not set state by state: it is (5) evaluated at the valuation of each state, and it rises because debt does not follow the price.

The size of both effects is set by how much leverage moves, and leverage moves only as much as the valuation ratio: its coefficient of variation is 0.12, against 0.02 in the baseline, where the ratio varies only in the states where admissibility binds. The channel is bounded by the volatility of valuations the model produces, which is the same quantity that leaves the model short of the data in Table 1.

6 Conclusion

This paper has replaced the constant debt-equity ratio of Barro and Jin (2021) with the state-dependent ratio that follows from fixing debt in units of consumption, leaving mean leverage and everything else in their model unchanged. The standard deviation of the levered excess return rises by 1.93% and the Sharpe ratio falls by 1.44%, while the mean premium rises by 0.47%. Measured against the distance between the model and the data, the two changes cover about 1% and 2% of the respective gaps.

The mechanism accounts for the sign and for the size. Leverage is high where the valuation ratio is low, and in this model those are the states where the excess return is most dispersed, so replacing the constant ratio raises the dispersion of the levered return by more than it raises its mean. The size is set by how much leverage moves, and leverage moves only as much as the valuation ratio: its coefficient of variation is 0.12.

The result is negative and quantified. This form of state-dependent leverage, well supported as it is by the evidence on capital structure, does not improve the model of Barro and Jin (2021) by any appreciable amount, and the reason lies in the small variability of the price-dividend ratio, which is the same shortcoming the authors themselves point to when they report that the model understates the volatility of returns and name the omission of a time-varying disaster probability as the main gap that remains. The order of work follows from this. The variability of valuations comes first, and only once that has been addressed can a leverage assumption of this kind produce an effect worth the name, since the direction in which it moves the second moments, small as it is here, is the right one.

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